| | |
|---|---|
| NAME ☞ | |
| SCHOOL ☞ | |
| | |
| TEACHER ☞ | |

# Pre-Leaving Certificate Examination, 2016

# Mathematics

## Paper 2

## Ordinary Level

## Time: 2 hours, 30 minutes

300 marks

| For examiner | |
|---|---|
| Question | Mark |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| Total | |

School stamp

| | |
|---|---|
| Running total | |

Grade

## Instructions

There are **two** sections in this examination paper.

| | | | |
|---|---|---|---|
| Section A | Concepts and Skills | 150 marks | 6 questions |
| Section B | Contexts and Applications | 150 marks | 3 questions |

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

**You will lose marks if all necessary work is not clearly shown.**

**You may lose marks if the appropriate units of measurement are not included, where relevant.**

**You may lose marks if your answers are not given in simplest form, where relevant.**

Write the make and model of your calculator(s) here: 

## Section A — Concepts and Skills — 150 marks

Answer **all six** questions from this section.

**Question 1** **(25 marks)**

A bag contains five €2 coins, three €1 coins and four 50c coins.
Coins are picked at random from the bag and not replaced.

**(a)** **(i)** Tom picks a coin from the bag. What is the probability that it is a €2 coin?

**(ii)** Tom then picks a second, third and fourth coin from the bag. What is the probability that all four coins are €2?

**(iii)** Find the probability that all four coins are **not** of the same value.

**(b)** **(i)** All of the coins are put back in the bag. Sarah picks four coins from the bag at random. What is the probability that she picks two €2 coins and then two 50c coins?

**(ii)** List all the ways in which Sarah can pick two €2 coins and two 50c coins and hence, find the probability that Sarah picks exactly two €2 and two 50c coins from the bag. For example, one possible way of picking these coins is shown below.

€2 €2 50c 50c

| page | running |
|---|---|
| | |

**Question 2** **(25 marks)**

A French teacher gave a written exam to two classes in the same year. The results of the examination for each class are shown in the table below.

| Class A | Class B |
|---|---|
| 36, 39, 45, 48, 49, 50, 53, 54, 54, 62, 63, 63, 64, 68, 69, 69, 71, 75, 77, 84, 88, 89, 92, 93, 97, 97 | 23, 25, 27, 34, 35, 35, 39, 42, 46, 48, 48, 49, 53, 54, 57, 57, 63, 64, 66, 68, 72, 73, 76, 81, 90 |

**(a)** Construct a back-to-back stem-and-leaf plot of the above data.

Key:

**(b)** **(i)** Find the median and interquartile range of the examination results in both classes.

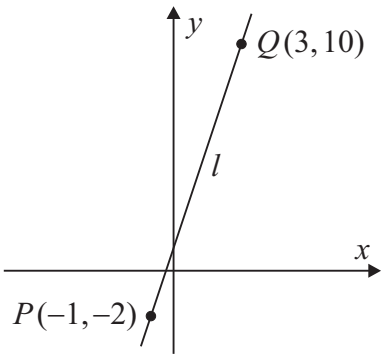
Median:

Interquartile range:

**(ii)** Describe what differences there are, if any, between the two distributions above.

**Question 3** **(25 marks)**

The line $l$ contains the points $P(-1, -2)$ and $Q(3, 10)$, as shown.




**(a)** Find the equation of $l$. Write your answer in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{Z}$.

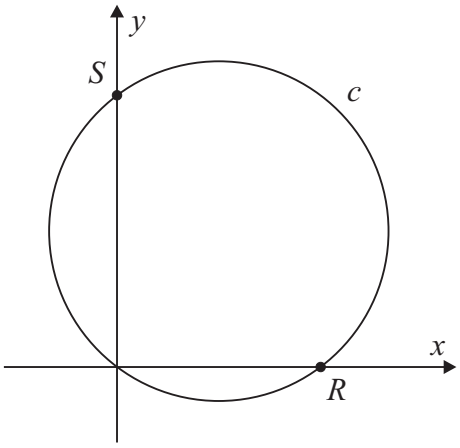
**(b)** The line $m$ passes through $Q$ and is perpendicular to $l$.
Find the equation of $m$.

**(c)** Hence, find the area of the triangle formed by $l$ and $m$ and the $y$-axis.

| page | running |
|---|---|
| | |

**Question 4** **(25 marks)**

The circle $c$ has centre (3, 4) and passes through the origin (0, 0), as shown.




**(a)** **(i)** Find the length of the radius of $c$.

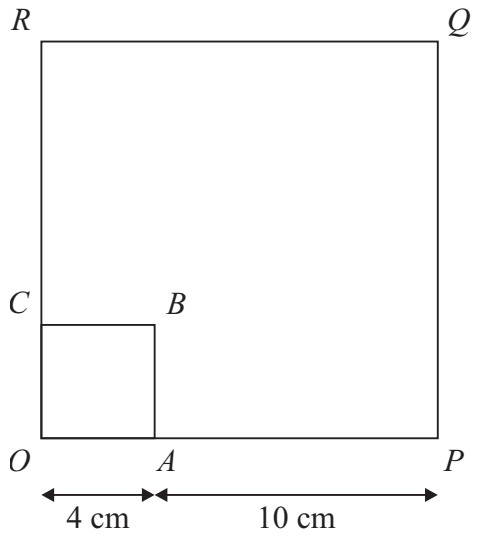
**(ii)** Hence, find the equation of $c$.

**(b)** $c$ intersects the $x$-axis at the point $R$ and the $y$-axis at the point $S$.
Calculate the co-ordinates of $R$ and $S$.

**(c)** Explain why $[RS]$ is a diameter of the circle $c$.

**Question 5** **(25 marks)**

The square $OPQR$ is the image of the square $OABC$ under an enlargement with centre $O$, as shown.
$|OA| = 4$ cm and $|AP| = 10$ cm.




**(a)** Find the scale factor of the enlargement.

**(b)** **(i)** Find $|AC|$ in the form $a\sqrt{2}$, where $a \in \mathbb{N}$.

**(ii)** Hence, use the scale factor to find $|PR|$.

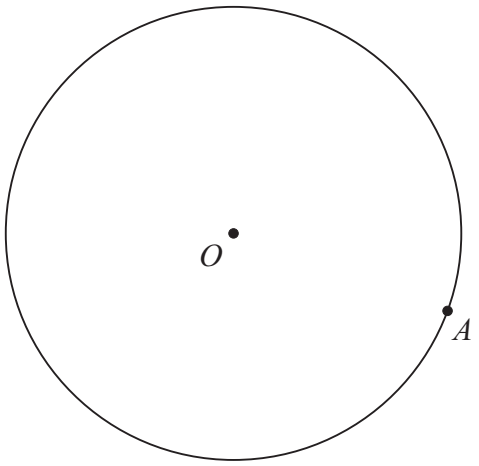
**(c)** Under another enlargement with centre $O$, the area of the image of the square $OABC$ is 81 cm$^2$. Find the scale factor of this enlargement.

| page | running |
|---|---|
| | |

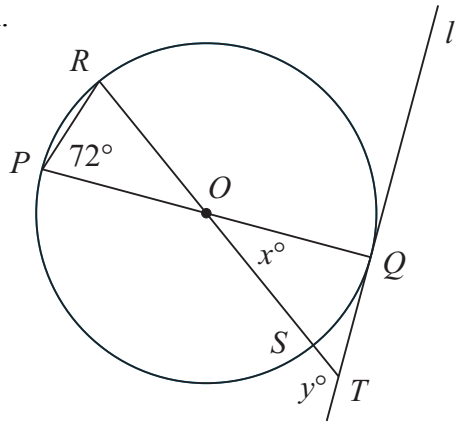
**Question 6** **(25 marks)**

**(a)** **(i)** Write down a geometric statement that can be used to construct a tangent to a circle at a point.

**(ii)** Construct the tangent to the circle below at the point $A$.
Show all your construction lines clearly.




**(b)** The line $l$ is a tangent to a circle at the point $Q$, as shown.
$[PQ]$ and $[RS]$ are diameters of the circle.
$[RS]$ is produced to intersect $l$ at the point $T$.




Find the value of $x$ and the value of $y$.
Give a reason for your answer in each case.

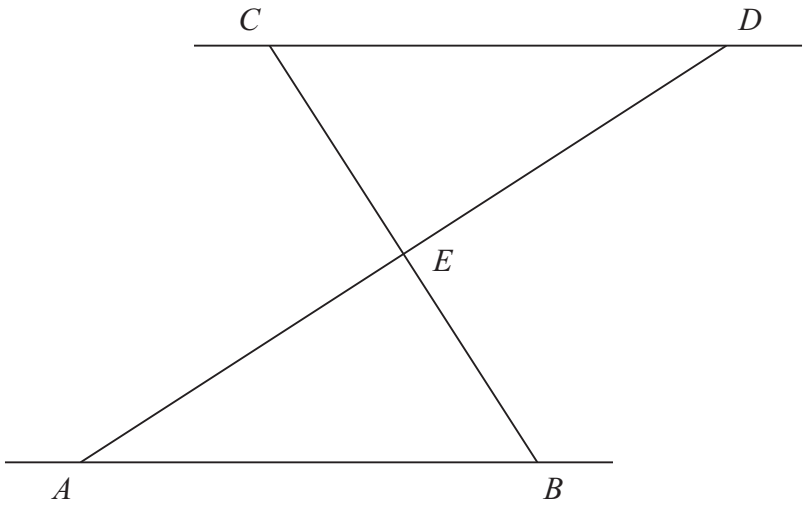
$x =$

Reason:

$y =$

Reason:

**(c)** The diagram shows two parallel lines, $AB$ and $CD$.
[ $AD$ ] bisects [ $CB$ ] at the point $E$.




Show that the triangles $ABE$ and $CDE$ are congruent.

| page | running |
|---|---|
| | |

## Section B — Contexts and Applications — 150 marks

Answer **all three** questions from this section.

**Question 7** **(60 marks)**

**(a)** Opinion polls are conducted regularly in Ireland to find out how people feel on a particular matter, such as what they think of a political party or issue.

**(i)** Find the margin of error of a survey, at 95% confidence, for each of the sample sizes in the table below. Write your answers as percentages, correct to one decimal place.

| Table 1 | | | | | |
|---|---|---|---|---|---|
| **Sample Size** | 20 | 100 | 500 | 1000 | 2000 |
| **Margin of Error** | | | | | |

**(ii)** Use the information in Table 1 to explain why the maximum sample size of most surveys is 1000 people.

**(b)** A *Red C* opinion poll was carried out in September 2015 to find out the first-preference voting intentions of people if there was a general election. The poll was conducted using a telephone survey of a random sample of 1005 adults eligible to vote. The table below shows the percentage of voters who were undecided and the percentage support for each of the main political groups when the undecided segment was excluded.

| Table 2 | |
|---|---|
| **Party / Group** | **Support** |
| Fine Gael | 28% |
| Labour | 10% |
| Fianna Fáil | 18% |
| Sinn Féin | 16% |
| Independents/Others | 28% |
| Undecided | 13% |

**(i)** By considering the margin of error in this survey, is it certain that Fianna Fáil are the second-largest political party? Justify your answer.

**(ii)** What other piece of information in Table 2 could give rise to a different overall level of support for Fianna Fáil in an actual election?

**(c)** A Fine Gael supporter claimed in a tweet that 30% of the population supported his party. Test this hypothesis using a 5% level of significance.

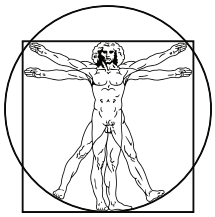
**(i)** State clearly the null hypothesis.

**(ii)** Create a 95% confidence interval for the level of support for Fine Gael, according to the opinion poll.

**(iii)** Is this sufficient evidence to accept the supporter's claim, at the 5% level of significance? Justify your answer.

| page | running |
|---|---|
| | |

**(d)** Leonardo da Vinci's 'Vitruvian Man' is a drawing which depicts a man in two superimposed positions inscribed in a circle and a square. The image displays the ideal human proportions of arm-span equal to height, as first described by the ancient Roman architect Vitruvius.



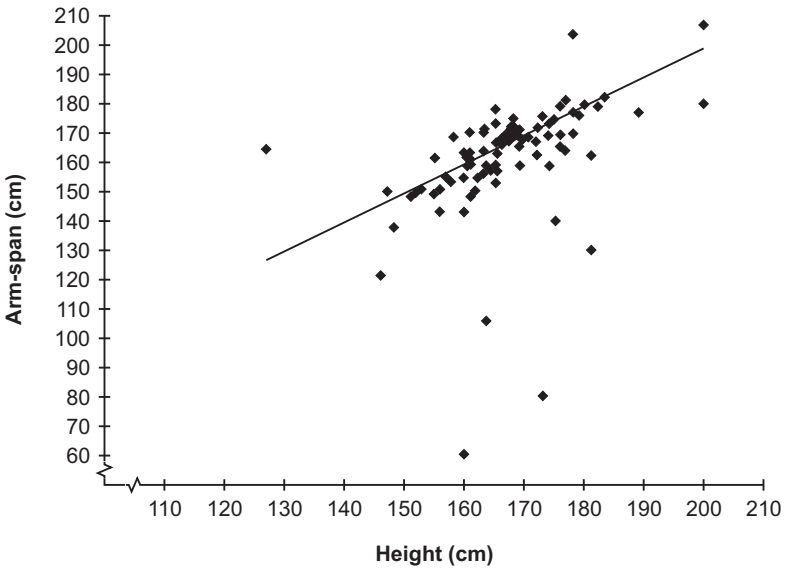
Gary wishes to investigate whether this hypothesis is valid over 500 years later. He carried out a small study at school by randomly selected 100 students and measured their height and arm-span.

Gary analysed the data using statistical software. He gets the software to produce the *summary statistics* shown in Table 3.

**Table 3**

| **Statistics** | **Height (cm)** | **Arm-span (cm)** |
|---|---|---|
| Mean | 166 | 162 |
| Minimum | 105 | 60 |
| Lower Quartile | 160 | 157 |
| Median | 165 | 165 |
| Upper Quartile | 173 | 171 |
| Maximum | 201 | 208 |
| Range | 96 | 148 |
| Interquartile Range | 13 | 14 |

Gary is interested in the relationship between height and arm-span. He produced the following scatter diagram of the data.




**(i)** Use the summary statistics in the table and the scatter diagram to complete the following sentences:

1. The height of the tallest student in the study is _____

2. The height of the student with the smallest arm-span is _____

3. The number of students with an arm-span between 145 cm and 185 cm is _____

**(ii)** Is a scatter plot the most suitable graphical method to illustrate the data that Gary has collected? Explain your answer.

**(iii)** The correlation coefficient between the height and arm-span of students in this study is one of the numbers below. Write the letter corresponding to the correct answer in the box.

| | |
|---|---|
| A | 0·82 |
| B | 0·58 |
| C | 0·22 |
| D | –0·22 |
| E | –0·58 |
| F | –0·82 |

**(iv)** What can you conclude from the scatter plot and the correlation coefficient?

**(v)** Circle an *outlier* on the diagram and write down the person's height and arm-span.

Height = Arm-span =

**(vi)** Give one possible issue that might make the results of this investigation unreliable. State clearly why the issue you mention could cause a problem.

| page | running |
|---|---|
| | |

**Question 8** **(50 marks)**

**(a)** The heights of all the members in a swim club were collected and recorded. The data was used to create the information shown in Table 1.

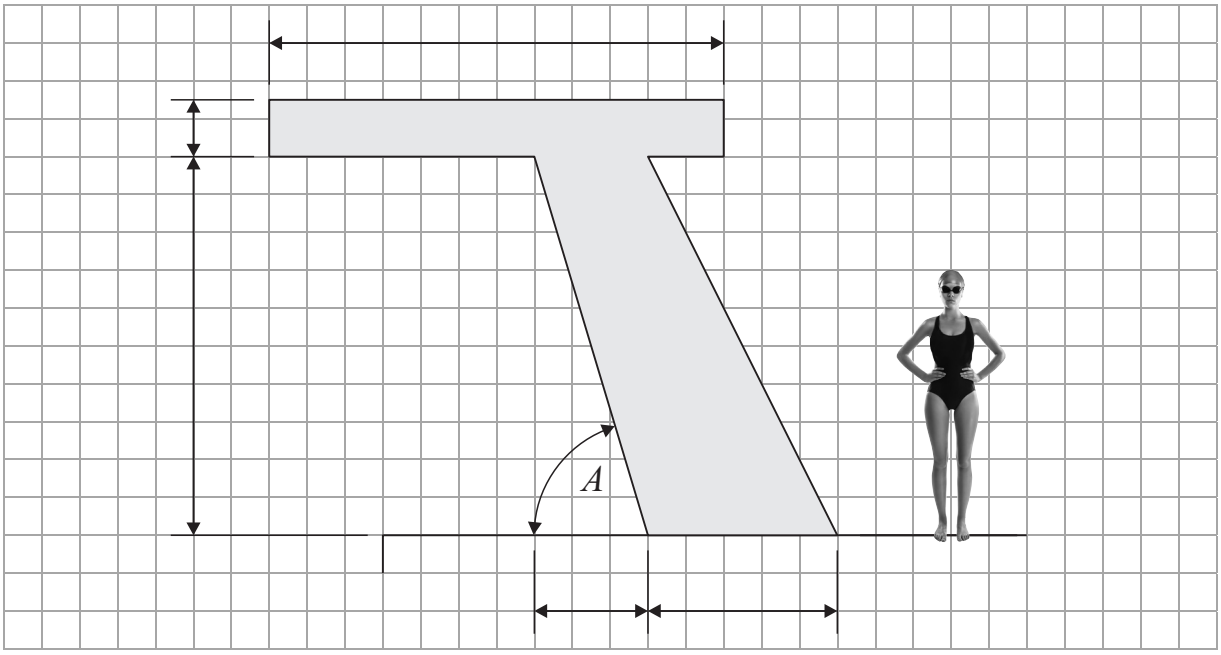
| **Table 1** | | | | |
|---|---|---|---|---|
| **Height (cm)** | 135-150 | 150-165 | 165-180 | 180-195 |
| **Number of members** | 4 | 8 | $x$ | 28 |

Taking mid-interval values, it was found that the mean height of members was 175 cm.

**(i)** Find the value of $x$ and hence, find the total number of members in the swim club.

**(ii)** What is the maximum number of members who could be smaller than the mean?

**(b)** The diagram below is a scale drawing of the diving platform extended over a swimming pool. The figure of the swimmer, who is of mean height in the swim club, standing beside the diving platform allows the scale of the drawing to be estimated.



**(i)** Using the mean height from part **(a)**, or otherwise, estimate the dimensions of the diving platform shown. Write your answers in the spaces provided on the diagram.

**(ii)** Using your answers in part **(i)** above, or otherwise, find the size of angle $A$, correct to the nearest degree.

**(iii)** Taking the width of the diving platform to be 1·5 m, find an estimate for the volume of concrete, in $m^3$, required to construct it.

**(c)** The swimming pool, to which the diving platform is attached, has a length of 30 m and a width of 20 m. The bottom of the pool is sloped, with a depth of 1·2 m at the shallow end and a depth of 4·5 m at the deep end.

**(i)** Find, in $m^3$, the volume of water required to fill the swimming pool.

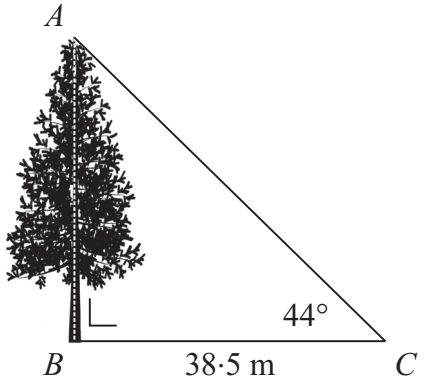
**(ii)** Given that the swimming pool is initially empty and water is pumped into the pool at the rate of 50 litres per second, how long does it take to fill the swimming pool? Give your answer in hours and minutes. (1000 litres = 1 $m^3$)

| page | running |
|---|---|
| | |

**Question 9** **(40 marks)**

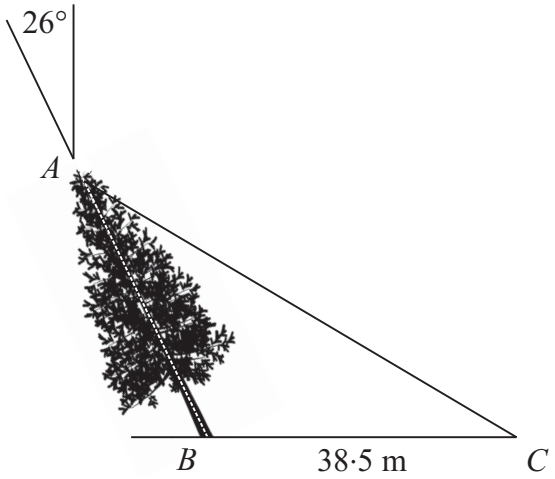
**(a)** The diagram shows a vertical tree, [ $AB$ ]. From a point $C$, which is on level ground 38·5 m from the base of the tree, $B$, the angle of elevation to the top of the tree, $A$, was measured as 44°.

Find | $AB$ |, correct to two decimal places.



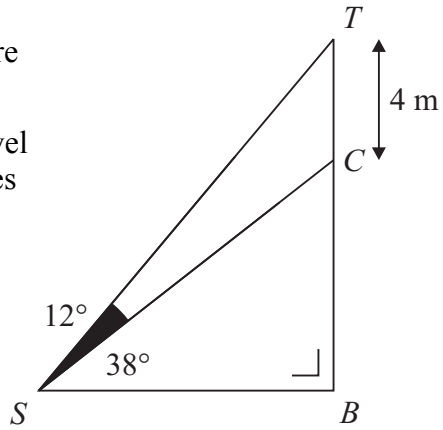

**(b)** The tree is damaged during a storm and it tilts 26° from the vertical axis, as shown. To stabilise the tree, a cable is attached to the tree 2 m from $A$ and the other end is pegged to the ground at $C$.

Find the length of the cable, correct to one decimal place.



**(c)** It is decided to reduce the height of a different tree by 4 m. In the diagram, [ $TB$ ] represents the height of the tree before the top of it is reduced from point $T$ to point $C$.

A tree surgeon stands at a point $S$, which is on the same level ground as $B$, the bottom of the tree, and measures the angles of elevation from this point to $T$ and $C$.
$|\angle CSB| = 38°$ and $|\angle TSC| = 12°$.




**(i)** Find $|\angle BTS|$.

**(ii)** Hence find $|SC|$, correct to two decimal places.

**(iii)** Find $|BC|$, the height of the tree after it is cut, correct to two decimal places.

| page | running |
|---|---|
| | |

You may use this page for extra work.

You may use this page for extra work.

| page | running |
|---|---|
| | |

Pre-Leaving Certificate, 2016 – Ordinary Level

# Mathematics – Paper 2

Time: 2 hours, 30 minutes