O. Murphy.

Teachers Manual

for

Fundamental Applied Mathematics

by

Oliver Murphy

Folens

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FOREWORD.

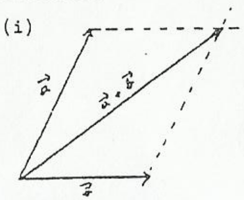
In this book the problems from the exercises of "Fundamental Applied Mathematics" are solved. In each case only one solution is given, even though there may be many ways of solving some of them. Secondly, these are not model solutions. The way in which they are written is to show briefly how each problem may be solved - not how questions should be answered in an exam.

O.M.

CHAPTER 1. VECTORS.

Exercise 1.A

1. (i)



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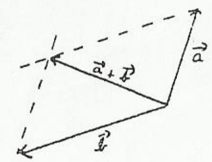

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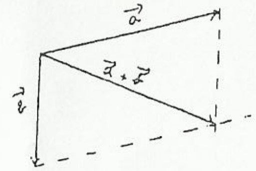

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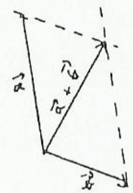

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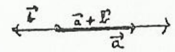
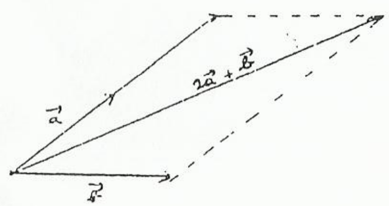


(vi) $\vec{0}$, the null vector

2.



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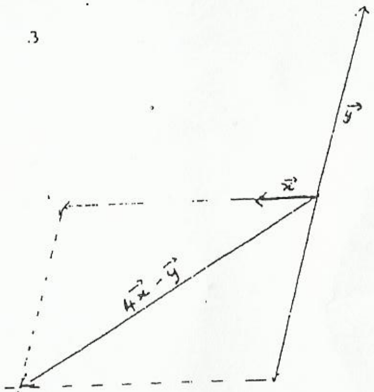

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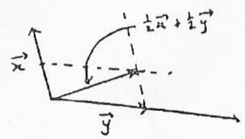

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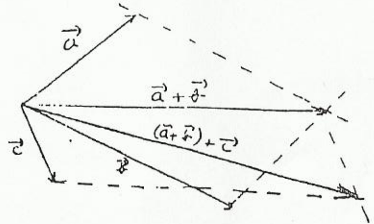

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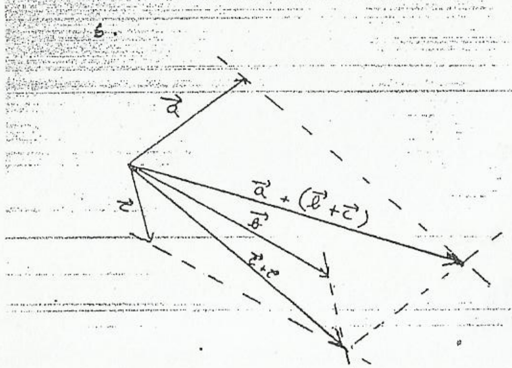
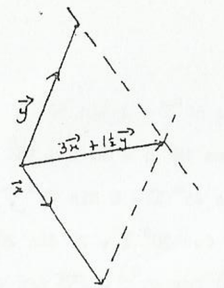


Yes; $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$.

7.



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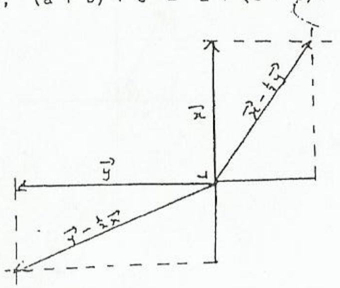


9. 5 cm; E 53° N.

10. Approximately 7 cm due East.

11. 13 cm.

Exercise 1.B

1. (i) $\sqrt{29}$, E 21° 48' N. (ii) $\sqrt{8}$, NE. (iii) 5, E 36° 52' S.
(iv) 13, W 67° 23' S. (v) $\sqrt{20}$, W 26° 34' N. (vi) $\sqrt{2}$, NE.
(vii) $\frac{1}{\sqrt{2}}$, SE. (viii) 1, W 53° 8' S. (ix) 2, W 30° N.
(x) $\sqrt{12}$, E 30° N. (xi) $\sqrt{48}$, E 60° S. (xii) 4, due West.

2. (i) $\vec{a} + \vec{b} = (3\vec{i} - \vec{j}) + (2\vec{i} - 3\vec{j}) = 5\vec{i} - 4\vec{j}$.
(ii) $\vec{a} - \vec{b} = (3\vec{i} - \vec{j}) - (2\vec{i} - 3\vec{j}) = \vec{i} + 2\vec{j}$.
(iii) $\vec{b} - \vec{a} = (2\vec{i} - 3\vec{j}) - (3\vec{i} - \vec{j}) = -\vec{i} - 2\vec{j}$.
(iv) $2\vec{a} - 3\vec{b} = 2(3\vec{i} - \vec{j}) - 3(2\vec{i} - 3\vec{j}) = 7\vec{j}$

3. $\vec{x} + \vec{y} = (2\vec{i} + 3\vec{j}) + (10\vec{i} + 2\vec{j}) = 12\vec{i} + 5\vec{j}$.
$|\vec{x} + \vec{y}| = \sqrt{12^2 + 5^2} = 13$.
$|\vec{x}| + |\vec{y}| = \sqrt{13} + \sqrt{104} = 3.606 + 10.20 = 13.806$.
$\therefore |\vec{x} + \vec{y}| < |\vec{x}| + |\vec{y}|$

4. $k(2\vec{i} - \vec{j}) + l(4\vec{i} + 3\vec{j}) = 2\vec{i} - 11\vec{j}$.
$\Rightarrow 2k + 4l = 2$ and $-k + 3l = -11$. Solving gives $l = -2$, $k = 5$.

5. (i) Magnitude $= \sqrt{3^2 + 4^2} = 5$. $\therefore \frac{1}{5}(3\vec{i} + 4\vec{j})$ has magnitude one.
(ii) $\frac{1}{\sqrt{5}}(\vec{i} + 2\vec{j})$ (ii) $\frac{1}{\sqrt{2}}(\vec{i} - \vec{j})$ (iii) $\frac{1}{\sqrt{10}}(-3\vec{i} - \vec{j})$ (iv) $\frac{1}{2}(\sqrt{3}\vec{i} + \vec{j})$.

6. $\vec{a} + \vec{b} = 4\vec{i} + 8\vec{j}$. $\therefore$ Slope $= \frac{j\text{-bit}}{i\text{-bit}} = \frac{8}{4} = 2$.
$\vec{a} - \vec{b} = 2\vec{i} - 6\vec{j}$. $\therefore$ Slope $= \frac{-6}{2} = -3$.
$m_1 \times m_2 = -6 \neq -1$. They are not perpendicular.

7. $m_1 = -\frac{1}{3}$, $m_2 = 3$. $m_1 \times m_2 = -1$. They are perpendicular.

8. $\vec{a} + t\vec{b} = (4\vec{i} - 2\vec{j}) + t(7\vec{i} + 5\vec{j}) = (4 + 7t)\vec{i} + (-2 + 5t)\vec{j}$.
$\vec{j}$-component = zero $\Rightarrow -2 + 5t = 0 \Rightarrow t = \frac{2}{5}$.

9. $\sqrt{8^2 + (-1)^2} = \sqrt{7^2 + k^2} \Rightarrow k^2 + 49 = 65 \Rightarrow k = \pm 4$.

10. $\sqrt{7^2 + (-1)^2} = \sqrt{p^2 + p^2} \Rightarrow 2p^2 = 50 \Rightarrow p = \pm 5$.

11. $m_1 = m_2 = -\frac{1}{3}$, therefore they are parallel.

12. $\frac{b}{a} \times \frac{d}{c} = -1 \Rightarrow bd = -ac \Rightarrow ac + bd = 0$.

13. $\frac{q}{p} = \frac{s}{r} \Rightarrow qr = ps$.

14. $\frac{p}{4} \times \frac{-2}{1} = -1 \Rightarrow -2p = -4 \Rightarrow p = 2$.

15. $\vec{u} = 10\text{Cos } 60^\circ \vec{i} + 10\text{Sin } 60^\circ \vec{j} = 5\vec{i} + 5\sqrt{3}\vec{j}$.

16. $\vec{v} = -2\text{Cos } 40^\circ \vec{i} + 2\text{Sin } 40^\circ \vec{j} = -1.53\vec{i} + 1.29\vec{j}$.

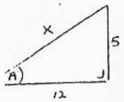
se 1.C

$|bc| = adj = H\cos\theta$; $|bc| = opp = H\sin\theta$.

$|ab| = opp = H\sin\theta$; $|bc| = adj = H\cos\theta$.

$|ac| = adj = H\cos\theta$; $|bc| = opp = H\sin\theta$.

$|ab| = opp = H\sin\theta$; $|ac| = adj = H\cos\theta$.

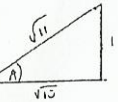


$\tan A = \frac{5}{12} = \frac{opp}{adj}$.

$x^2 = 5^2 + 12^2 = 169 \Rightarrow x = 13$.

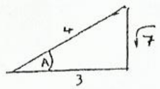
$\therefore \cos A = \frac{adj}{hyp} = \frac{12}{13}$. $\sin A = \frac{opp}{hyp} = \frac{5}{13}$.

i)



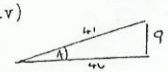
$\cos A = \sqrt{10}/\sqrt{11}$; $\sin A = 1/\sqrt{11}$.

i)



$\sin A = \sqrt{7}/4$; $\tan A = \sqrt{7}/3$

v)



$\cos A = 40/41$.

) $|ab| = adj = 8\cos 30^\circ = 8(\sqrt{3}/2) = 4\sqrt{3}$ cm.

$|bc| = opp = 8\sin 30^\circ = 8(\frac{1}{2}) = 4$ cm.

ii) $|xy| = \sqrt{8}\cos 45^\circ = 2$ m ; $|yz| = \sqrt{8}\sin 45^\circ = 2$ m.

ii) $|ab| = 10\sin 40^\circ = 6.428$ m ; $|bc| = 10\cos 40^\circ = 7.66$ m.

iv) $|xy| = 20\cos 35^\circ = 16.38$ cm ; $|xz| = 20\sin 35^\circ = 11.47$ cm.

(v) $|pq| = 40\cos 20^\circ = 37.59$ m ; $|qr| = 40\sin 20^\circ = 13.68$ ...

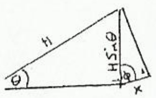
vi) $|pq| = 12\cos 60^\circ = 6$ m ; $|qr| = 12\sin 60^\circ = 6\sqrt{3}$ m.

ii) $|ab| = 15\cos\theta = 15(\frac{4}{5}) = 12$ cm ; $|bc| = 15\sin\theta = 15(\frac{3}{5}) = 9$ cm.

ii) $|pq| = 78\sin\alpha = 78(\frac{12}{13}) = 72$ m ; $|qr| = 78\cos\alpha = 78(\frac{5}{13}) = 30$ m.

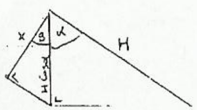
ix) $|xy| = \sqrt{13}\cos\alpha = \sqrt{13}(\frac{3}{\sqrt{13}}) = 3$; $|yz| = \sqrt{13}\sin\alpha = \sqrt{13}(\frac{2}{\sqrt{13}}) = 2$.

(x) $|ab| = \sqrt{20}\cos\alpha = \sqrt{20}(\frac{2}{\sqrt{5}}) = 4$; $|bc| = \sqrt{20}\sin\alpha = \sqrt{20}(\frac{1}{\sqrt{5}}) = 2$.



$X = ADJ = H\sin\theta\cos\phi$.

$\therefore X = (13)(\frac{3}{5})(\frac{5}{13}) = 3$.



$X = ADJ = H\cos\alpha\cos\beta$.

$\therefore X = H(\frac{7}{\sqrt{50}})(\frac{1}{\sqrt{2}}) = \frac{7}{10}H$. $\therefore H : X = 10 : 7$.

Exercise 1.D

1. (i) $2\cos 60^\circ\vec{i} + 2\sin 60^\circ\vec{j} = \vec{i} + \sqrt{3}\vec{j}$.

(ii) $10\cos 18^\circ\vec{i} + 10\sin 18^\circ\vec{j} = 9.511\vec{i} + 3.09\vec{j}$.

(iii) $8\cos 45^\circ\vec{i} - 8\sin 45^\circ\vec{j} = 4\sqrt{2}\,\vec{i} - 4\sqrt{2}\,\vec{j}$.

(iv) $-20\cos 20^\circ\vec{i} + 20\sin 20^\circ\vec{j} = -18.794\vec{i} + 6.84\vec{j}$.

(v) $-\sqrt{50}\cos 45^\circ\vec{i} - \sqrt{50}\sin 45^\circ\vec{j} = -5\vec{i} - 5\vec{j}$.

(vi) $12\cos 39^\circ\vec{i} - 12\sin 39^\circ\vec{j} = 9.3252\vec{i} - 7.5516\vec{j}$.

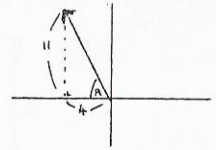
2. $\vec{u} = -10\cos\alpha\,\vec{i} - 10\sin\alpha\,\vec{j} = -8\vec{i} - 6\vec{j}$.

$\vec{v} = 13\cos\beta\,\vec{i} - 13\sin\beta\,\vec{j} = 12\vec{i} - 5\vec{j}$.

$\therefore \vec{u} + \vec{v} = 4\vec{i} - 11\vec{j}$. $\vec{w} = -(\vec{u} + \vec{v}) = -4\vec{i} + 11\vec{j}$.

$|\vec{w}| = \sqrt{(-4)^2 + (11)^2} = \sqrt{137} = 11.7$

$\tan A = \frac{11}{4} \Rightarrow A = 70^\circ$. Direction is W 70° N.



3. $\vec{p} = \sqrt{8}\cos 45^\circ\vec{i} + \sqrt{8}\sin 45^\circ\vec{j} = 2\vec{i} + 2\vec{j}$.

$\vec{q} = 4\cos 30^\circ\vec{i} - 4\sin 30^\circ\vec{j} = 2\sqrt{3}\vec{i} - 2\vec{j}$.

$\therefore \vec{p} + \vec{q} = (2 + 2\sqrt{3})\vec{i} + 0\vec{j}$.

4. $\vec{r} = -10\cos 40^\circ\vec{i} - 10\sin 40^\circ\vec{j} = -7.66\vec{i} - 6.428\vec{j}$.

$\vec{s} = -10\cos 58^\circ\vec{i} + 10\sin 58^\circ\vec{j} = -5.299\vec{i} + 8.48\vec{j}$.

$\vec{t} = 11\cos 20^\circ\vec{i} + 11\sin 20^\circ\vec{j} = 10.3367\vec{i} + 3.762\vec{j}$.

$\therefore \vec{r} + \vec{s} + \vec{t} = -2.6\vec{i} + 5.8\vec{j}$.

5. $\vec{a} = 12\vec{i}$. $\vec{b} = -13\cos\alpha\,\vec{i} + 13\sin\alpha\,\vec{j} = -12\vec{i} + 5\vec{j}$.

$\therefore \vec{a} + \vec{b} = 12\vec{i} - 12\vec{i} + 5\vec{j} = 5\vec{j}$, along the $\vec{j}$-axis. $|\vec{a} + \vec{b}| = 5$ units.

6. $\vec{x} = -25\cos A\,\vec{i} + 25\sin A\,\vec{j} = -20\vec{i} + 15\vec{j}$.

$y = 17\cos B\,\vec{i} - 17\sin B\,\vec{j} = 8\vec{i} - 15\vec{j}$.

$\therefore \vec{x} + \vec{y} = -20\vec{i} + 15\vec{j} + 8\vec{i} - 15\vec{j} = -12\vec{i}$, which has no $\vec{j}$-component.

$|\vec{x} + \vec{y}| = 12$ units.

Exercise 1.E

1. $\vec{a} = 10\vec{i}$. $\quad \vec{b} = -26\text{ Cos}\alpha\,\vec{i} + 26\text{ Sin}\alpha\,\vec{j}$.

$\therefore\ \vec{a} + \vec{b} = (10 - 26\text{ Cos}\alpha)\vec{i} + 26\text{ Sin}\alpha\,\vec{j}$.

No $\vec{i}$-component $\Rightarrow$ $10 - 26\text{ Cos}\alpha = 0$ $\Rightarrow$ $\text{Cos}\alpha = \frac{5}{13}$ $\Rightarrow$ $\text{Sin}\alpha = \frac{12}{13}$.

$\therefore\ \vec{a} + \vec{b} = 0\vec{i} + 26(\frac{12}{13})\,\vec{j} = 24\vec{j}$. $\quad \therefore |\vec{a} + \vec{b}| = 24$ units.

2. $\vec{a} = \sqrt{32}(\frac{1}{\sqrt{2}})\vec{i} - \sqrt{32}(\frac{1}{\sqrt{2}})\vec{j} = 4\vec{i} - 4\vec{j}$.

$\vec{b} = -5\text{ Cos}\alpha\,\vec{i} + 5\text{ Sin}\alpha\,\vec{j}$.

$\therefore\ \vec{a} + \vec{b} = (4 - 5\text{ Cos}\alpha)\vec{i} + (-4 + 5\text{ Sin}\alpha)\vec{j}$.

No $\vec{i}$-component $\Rightarrow$ $4 - 5\text{ Cos}\alpha = 0$ $\Rightarrow$ $\text{Cos}\alpha = \frac{4}{5}$ $\Rightarrow$ $\text{Sin}\alpha = \frac{3}{5}$.

$\therefore\ \vec{a} + \vec{b} = 0\vec{i} + (-4 + 5(\frac{3}{5}))\vec{j} = 0\vec{i} - \vec{j}$: Answer.

3. Let $|\vec{b}| = X$.

$\vec{a} = -4\vec{i}$. $\quad \vec{b} = \frac{1}{2}X\vec{i} + \frac{\sqrt{3}}{2}X\vec{j}$.

$\therefore\ \vec{a} + \vec{b} = (-4 + \frac{1}{2}X)\vec{i} + \frac{\sqrt{3}}{2}X\vec{j}$.

No $\vec{i}$-component $\Rightarrow$ $-4 + \frac{1}{2}X = 0$ $\Rightarrow$ $X = 8$. i.e. $|\vec{b}| = 8$.

$\therefore\ \vec{a} + \vec{b} = 0\vec{i} + 4\sqrt{3}\vec{j}$. $\quad |\vec{a} + \vec{b}| = 4\sqrt{3}$ units.

4. $\vec{r} = -10\text{ Cos }70^\circ\vec{i} + 10\text{ Sin }70^\circ\vec{j} = -3.420\,\vec{i} + 9.397\,\vec{j}$.

Let $|\vec{s}| = X$ $\Rightarrow$ $\vec{s} = X\vec{i}$.

$\therefore\ \vec{r} + \vec{s} = (-3.420 + X)\vec{i} + 9.397\,\vec{j}$

No $\vec{i}$-component $\Rightarrow$ $X = 3.420$. i.e. $|\vec{s}| = 3.420$.

5. $\vec{a} = 10\text{ Cos }\theta\,\vec{i} + 10\text{ Sin }\theta\,\vec{j} = 10(\frac{4}{5})\vec{i} + 10(\frac{3}{5})\vec{j} = 8\vec{i} + 6\vec{j}$.

Let $|\vec{b}| = X$ $\Rightarrow$ $\vec{b} = -X\vec{j}$

$\therefore\ \vec{a} + \vec{b} = 8\vec{i} + (6 - X)\vec{j} = k\vec{i}$.

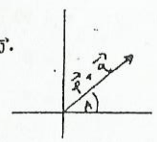
$\therefore\ X = 6$ and $k = 8$.

6. $\vec{p} = 35\text{ Cos}\alpha\,\vec{i} + 35\text{ Sin}\alpha\,\vec{j} = 35(\frac{3}{5})\vec{i} + 35(\frac{4}{5})\vec{j} = 21\vec{i} + 28\vec{j}$.

$q = 13\text{ Cos}\beta\,\vec{i} + 13\text{ Sin}\beta\,\vec{j} = 13(\frac{12}{13})\vec{i} + 13(\frac{5}{13})\vec{j} = 12\vec{i} + 5\vec{j}$.

$\therefore\ \vec{p} + \vec{q} = 21\vec{i} + 28\vec{j} + 12\vec{i} + 5\vec{j} = 33\vec{i} + 33\vec{j}$.

$\therefore\ \text{Tan } A = \frac{33}{33} = 1$ $\Rightarrow$ $A = 45^\circ$.



7. $r = -3.42\,\vec{i} + 9.397\,\vec{j}$.

Let $|\vec{s}| = X$. $\quad \therefore s = X\text{ Cos }10^\circ\vec{i} + X\text{ Sin }10^\circ\vec{j} = 0.9848X\,\vec{i} + 0.1736X\,\vec{j}$.

$\therefore\ \vec{r} + \vec{s} = (-3.42 + 0.9848X)\vec{i} + (9.397 + 0.1736X)\vec{j}$.

Since $(\vec{r} + \vec{s})$ is in a NE direction, the $\vec{i}$ and $\vec{j}$ components must be equal.

$\therefore\ -3.42 + 0.9848X = 9.397 + 0.1736X$ $\Rightarrow$ $X = 15.8$: Answer.

8. $|\vec{p}| = \sqrt{8^2 + 1^2} = \sqrt{65}$. $\quad |\vec{q}| = \sqrt{(-7)^2 + 4^2} = \sqrt{65}$. $\quad \therefore |\vec{p}| = |\vec{q}|$.

"Slope" of p $= \frac{\vec{j}\text{-bit}}{\vec{i}\text{-bit}} = \frac{1}{8} = m_1$.

"Slope" of q $= \frac{4}{-7} = -\frac{4}{7} = m_2$.

$\text{Tan }\theta = \pm\frac{m_1 - m_2}{1 + m_1m_2} = \pm\frac{1/8 + 4/7}{1 - 1/14} = \pm\frac{3}{4}$.

θ is obtuse, therefore $\text{Tan }\theta = -\frac{3}{4}$ $\Rightarrow$ $\theta = 143^\circ$.

9. $|-7\vec{i} + \vec{j}| = \sqrt{49 + 1} = \sqrt{50}$.

$|p(\vec{i} + \vec{j})| = |p\vec{i} + p\vec{j}| = \sqrt{p^2 + p^2} = \sqrt{2p^2}$.

$\therefore\ 2p^2 = 50$ $\Rightarrow$ $p = 5$ (since $p > 0$).

$m_1 = -1/7$. $\quad m_2 = 5/5 = 1$.

$\text{Tan } A = \pm\frac{m_1 - m_2}{1 + m_1m_2} = \pm\frac{-1/7 - 1}{1 - 1/7} = \pm\frac{4}{3}$.

$\text{Tan } A = -1.3333$ (since A is obtuse.)

$\therefore\ A = 126^\circ\ 52'$.

10. $\vec{a} = 10\text{ Cos }80^\circ\,\vec{i} + 10\text{ Sin }80^\circ\,\vec{j} = 1.736\,\vec{i} + 9.848\,\vec{j}$.

Let $|\vec{b}| = X$. Therefore, $\vec{b} = X\vec{i}$.

$\therefore\ \vec{a} + \vec{b} = (1.736 + X)\vec{i} + 9.848\vec{j}$.

Since the direction of $(\vec{a} + \vec{b})$ is $21^\circ\ 48'$, the slope of $(\vec{a} + \vec{b})$ must equal Tan $21^\circ 48'$.

$\therefore\ \frac{9.848}{1.736 + X} = 0.4$ $\Rightarrow$ $X = 22.88$: Answer.

Exercise 2.A

1. (i)
$$\left.\begin{array}{l} u = 0 \\ v = 10 \\ t = 5 \\ a = ? \end{array}\right\} \quad \begin{array}{rl} & v = u + at \\ \Rightarrow & 10 = 0 + 5a \\ \Rightarrow & a = 2 \text{ m/s}^2. \end{array}$$

(ii)
$$\left.\begin{array}{l} u = 0 \\ a = 2 \\ t = 5 \\ s = ? \end{array}\right\} \quad \begin{array}{rl} & s = ut + \tfrac{1}{2}at^2 \\ \Rightarrow & s = (0)(5) + \tfrac{1}{2}(2)(25) = 25 \text{ m}. \end{array}$$

2. (i)
$$\left.\begin{array}{l} u = 0 \\ v = 24 \\ a = 3 \\ t = ? \end{array}\right\} \quad \begin{array}{rl} & v = u + at \\ \Rightarrow & 24 = 0 + 3t \\ \Rightarrow & t = 8 \text{ s}. \end{array}$$

(ii)
$$\left.\begin{array}{l} u = 0 \\ a = 3 \\ t = 8 \\ s = ? \end{array}\right\} \quad \begin{array}{rl} & s = ut + \tfrac{1}{2}at^2 \\ \Rightarrow & s = (0)(8) + \tfrac{1}{2}(3)(64) = 96 \text{ m}. \end{array}$$

3.
$$\left.\begin{array}{l} u = 0 \\ a = 3 \\ s = 6 \\ v = ? \end{array}\right\} \quad \begin{array}{rl} & v^2 = u^2 + 2as \\ \Rightarrow & v^2 = 0 + 2(3)(6) \\ \Rightarrow & v = 6 \text{ m/s}. \end{array}$$

4.
$$\left.\begin{array}{l} u = 50 \\ v = 70 \\ s = 300 \\ a = ? \end{array}\right\} \quad \begin{array}{rl} & v^2 = u^2 + 2as \\ \Rightarrow & 4900 = 2500 + 2(a)(300) \\ \Rightarrow & a = 4 \text{ m/s}^2. \end{array}$$

$$\left.\begin{array}{l} u = 50 \\ v = 70 \\ a = 4 \\ t = ? \end{array}\right\} \quad \begin{array}{rl} & v = u + at \\ \Rightarrow & 70 = 50 + 4t \\ \Rightarrow & t = 5 \text{ s}. \end{array}$$

5. (i)
$$\left.\begin{array}{l} u = 24 \\ v = 0 \\ a = -8 \\ s = ? \end{array}\right\} \quad \begin{array}{rl} & v^2 = u^2 + 2as \\ \Rightarrow & 0 = 576 + 2(-8)s \\ \Rightarrow & s = 36 \text{ m}. \end{array}$$

(ii)
$$\begin{array}{rl} & v^2 = u^2 + 2as \\ \Rightarrow & 0 = 2304 + 2(-8)s \\ \Rightarrow & s = 144 \text{ m}. \end{array}$$

6.
$$\left.\begin{array}{l} u = 80 \\ v = 50 \\ s = 650 \\ a = ? \end{array}\right\} \quad \begin{array}{rl} & v^2 = u^2 + 2as \\ \Rightarrow & 2500 = 6400 + 2a(650) \\ \Rightarrow & a = -3 \text{ m/s}^2. \end{array}$$

Answer: The deceleration is 3 m/s^2.

7.
$$\left.\begin{array}{l} u = 3 \\ v = 11 \\ t = 6 \\ s = ? \end{array}\right\} \quad \begin{array}{rl} & s = \left(\frac{u + v}{2}\right) t \\ \Rightarrow & s = \left(\frac{3 + 11}{2}\right)(6) \\ & = 42 \text{ m}. \end{array}$$

8.
$$\left.\begin{array}{l} u = 3 \\ v = 0 \\ s = 6 \\ a = ? \end{array}\right\} \quad \begin{array}{rl} & v^2 = u^2 + 2as \\ \Rightarrow & 0 = 9 + 2a(6) \\ \Rightarrow & a = -\tfrac{3}{4} \text{ m/s}^2. \end{array}$$

$$\left.\begin{array}{l} u = 3 \\ v = 0 \\ a = -\tfrac{3}{4} \\ t = ? \end{array}\right\} \quad \begin{array}{rl} & v = u + at \\ \Rightarrow & 0 = 3 + (-\tfrac{3}{4})t \\ \Rightarrow & t = 4 \text{ s}. \end{array}$$

9.
$$\left.\begin{array}{l} u = 70 \\ v = 50 \\ t = 8 \\ a = ? \end{array}\right\} \quad \begin{array}{rl} & v = u + at \\ \Rightarrow & 70 = 50 + a(8) \\ \Rightarrow & a = -2\tfrac{1}{2} \text{ m/s}^2. \end{array}$$

$$\left.\begin{array}{l} u = 70 \\ t = 8 \\ a = -2\tfrac{1}{2} \\ s = ? \end{array}\right\} \quad \begin{array}{rl} & s = ut + \tfrac{1}{2}at^2 \\ \Rightarrow & s = 70(8) + \tfrac{1}{2}(-2\tfrac{1}{2})(64) \\ & = 560 - 80 = 480 \text{ m}. \end{array}$$

9. (continued) $\left.\begin{array}{l} u = 50 \\ v = 0 \\ a = -2\frac{1}{2} \\ s = ? \end{array}\right\}$ $\quad v^2 = u^2 + 2as$

$\Rightarrow \quad 0 = 2500 + 2(-2\frac{1}{2})s$

$\Rightarrow \quad s = 500$ m.

10. (a) $72 \text{ km/hr} = \dfrac{72000 \text{ m}}{3600 \text{ s}} = 20$ m/s.

(b) $48 \text{ km/hr} = \dfrac{48000 \text{ m}}{3600 \text{ s}} = \dfrac{40}{3}$ m/s.

$v^2 = u^2 + 2as \Rightarrow \dfrac{1600}{9} = 400 + 2a(500) \Rightarrow a = -\dfrac{2}{9}$ m/s^2

$v = u + at \Rightarrow \dfrac{40}{3} = 20 + (-\dfrac{2}{9})t \Rightarrow t = 30$ s.

$v^2 = u^2 + 2as \Rightarrow 0 = \dfrac{1600}{9} + 2(-\dfrac{2}{9})s \Rightarrow s = 400$ m.

11. $1 \text{ km/hr} = \dfrac{1000 \text{ m}}{3600 \text{ s}} = \dfrac{5}{18}$ m/s.

$\therefore$ 72 km/hr = 20 m/s and 54 km/hr = 15 m/s.

$v^2 = u^2 + 2as \Rightarrow 225 = 400 + 2(a)(35) \Rightarrow a = -2\frac{1}{2}$ m/s^2.

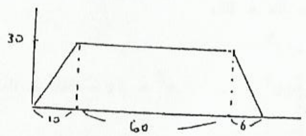
$v^2 = u^2 + 2as \Rightarrow 0 = 225 + 2(-2\frac{1}{2})s \Rightarrow s = 45$ m.

Exercise 2.B

1. $v = u + at \Rightarrow 30 = 0 + 3t \Rightarrow t = 10$ s.

$v = u + at \Rightarrow 0 = 30 + (-3)t \Rightarrow t = 6$ s.

(ii)



s = area under the curve

$= \frac{1}{2}(10)(30) + (60)(30) + \frac{1}{2}(6)(30)$

$= 2040$ m.

2.(i) First part: $s = (\dfrac{u + v}{2})t = (\dfrac{0 + 24}{2})(12) = 144$ m.

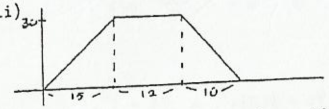
Second part: $s = (\dfrac{u + v}{2})t = (\dfrac{24 + 0}{2})(3) = 36$ m.

Total distance = 144 + 36 = 180 m.

(ii) Average speed $= \dfrac{\text{total distance}}{\text{total time}} = \dfrac{180}{15} = 12$ m/s.

3. (i) $v = u + at \Rightarrow 30 = 0 + 2t \Rightarrow t = 15$ s.

(ii)



Distance = Area under the curve

$= \frac{1}{2}(15)(30) + (12)(30) + \frac{1}{2}(10)(30)$

$= 735$ m.

(iii) $v = u + at \Rightarrow 0 = 30 + a(10) \Rightarrow a = -3$ m/s^2.

Answer: The deceleration is 3 m/s^2.

4. (i) $v = u + at \Rightarrow 20 = 0 + a(5) \Rightarrow a = 4$ m/s^2.

(ii) $s = ut + \frac{1}{2}at^2 \Rightarrow s = 0(5) + \frac{1}{2}(4)(25) = 50$ m.

(iii) Remaining distance = 200 m. 200 m at 20 m/s takes 10 seconds.

The total time taken = 5 + 10 = 15 s.

5. (i) $v^2 = u^2 + 2as \Rightarrow (40)^2 = 0^2 + 2(2)s \Rightarrow s = 400$ m.

$v = u + at \Rightarrow 40 = 0 + 2t \Rightarrow t = 20$ s.

(ii) $v^2 = u^2 + 2as \Rightarrow 0 = (40)^2 + 2(-5)s \Rightarrow s = 160$ m.

$v = u + at \Rightarrow 0 = 40 + (-5)t \Rightarrow t = 8$ s.

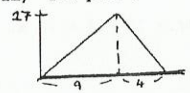
(iii) Remainder = 1000 − 400 − 160 = 440 m.

440 m at 40 m/s takes 11 seconds. $\therefore$ Total time = 20 + 11 + 8 = 39 s.

6. (i) $v = u + at \Rightarrow 27 = 0 + a(9) \Rightarrow a = 3$ m/s^2.

(ii) $v^2 = u^2 + 2as \Rightarrow 0 = (27)^2 + 2(a)(54) \Rightarrow a = -6\frac{3}{4}$ m/s^2.

(iii) 2nd part : $v = u + at \Rightarrow 0 = 27 + (-6\frac{3}{4})t \Rightarrow t = 4$ s.



Distance = Area under the curve

$= \frac{1}{2}(13)(27) = 175\frac{1}{2}$ m

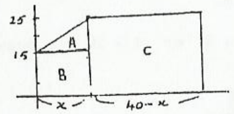
Average speed $= \dfrac{\text{Total distance}}{\text{Total time}} = \dfrac{175\frac{1}{2}}{13} = 13\frac{1}{2}$ m/s.

(iv) First part : $v = u + at \Rightarrow 15 = 0 + 3t \Rightarrow t = 5$ s

2nd part : $v = u + at \Rightarrow 15 = 27 + (-6\frac{3}{4})t \Rightarrow t = 1\frac{7}{9}$ s.

Answer: After 5 seconds and after $(9 + 1\frac{7}{9}) = 10\frac{7}{9}$ seconds.

7.



Area under the curve = 980

$\Rightarrow \frac{1}{2}(x)(10) + (15)(x) + (40 - x)(25) = 980$

$\Rightarrow x = 4$ seconds.

$v = u + at \Rightarrow 25 = 15 + (a)(4) \Rightarrow a = 2\frac{1}{2}$ m/s^2.

8. 30 km/hr $= \frac{30000 \text{ m}}{3600 \text{ s}} = \frac{25}{3}$ m/s.

$v = u + at \Rightarrow \frac{25}{3} = 0 + 2t \Rightarrow t = 4\frac{1}{6}$ seconds.

$v^2 = u^2 + 2as \Rightarrow (\frac{50}{3})^2 = 0^2 + 2(2)s \Rightarrow s = \frac{625}{9}$ m.

$v^2 = u^2 + 2as \Rightarrow 0^2 = (\frac{50}{3})^2 + 2(a)(2) \Rightarrow a = -\frac{625}{9}$ m/s^2.

9. Let t be the time of meeting.

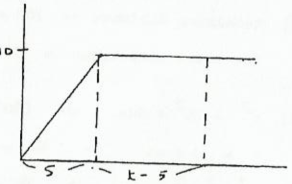
$s_1 = 5t + \frac{1}{2}(3)t^2 = 5t + \frac{3}{2}t^2$. $s_2 = 7t + \frac{1}{2}(2)t^2 = 7t + t^2$.

$s_1 + s_2 = 162 \Rightarrow 12t + \frac{5}{2}t^2 = 162 \Rightarrow t = 6.$ ($t = -\frac{54}{5}$ rejected)

At $t = 6$, $v_1 = u + at = 5 + (3)(6) = 23$ m/s ; $v_2 = 7 + (2)(6) = 19$ m/s.

Exercise 2.C

1. $v = u + at \Rightarrow 10 = 0 + 2t \Rightarrow t = 5$ s.



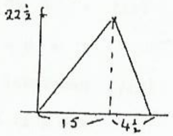
Area under the curve = 100

$\Rightarrow \frac{1}{2}(5)(10) + (t - 5)(10) = 100$

$\Rightarrow t = 12.5$ seconds.

2. $v = u + at \Rightarrow v = 0 + (1\frac{1}{2})(15) = 22\frac{1}{2}$ m/s.

$v = u + at \Rightarrow 0 = 22\frac{1}{2} + (-5)t \Rightarrow t = 4\frac{1}{2}$ s.



Distance = Area under the curve $= \frac{1}{2}(19\frac{1}{2})(22\frac{1}{2}) = 219\frac{3}{8}$ m.

3. (i) $v = u + at \Rightarrow 24 = 0 + 2t \Rightarrow t = 12$ s.

$s = (\frac{u+v}{2})t \Rightarrow 48 = (\frac{24+0}{2})t \Rightarrow t = 4$ s.

(ii) First part : $s = ut + \frac{1}{2}at^2 = 0(12) + \frac{1}{2}(2)(12)^2 = 144$ m.

$\therefore$ Total distance = 144 + 48 = 192 m. Total time = 12 + 4 = 16 s.

$\therefore$ Average speed $= \frac{192}{16} = 12$ m/s.

4. $s_1 = ut + \frac{1}{2}at^2 = 0(t) + \frac{1}{2}(4)t^2 = 2t^2$. $s_2 = 20t$.

$s_1 = s_2 \Rightarrow 2t^2 = 20t \Rightarrow t = 10$ s $\Rightarrow s = 200$ m.

5. (i)$v_1 = 10 + 3t$; $v_2 = 20 + 2t$. (ii) $s_1 = 10t + \frac{3}{2}t^2$; $s_2 = 20t + t^2$.

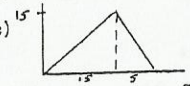
(a) $v_1 = v_2 \Rightarrow 10 + 3t = 20 + 2t \Rightarrow t = 10$ s.

(b) $s_1 = s_2 \Rightarrow 10t + \frac{3}{2}t^2 = 20t + t^2 \Rightarrow t = 0$ or $t = 20$ s : Answer.

6. (a) $t_1 : t_2 = 3 : 1 = \frac{3}{4} : \frac{1}{4}$. $\therefore t_1 = \frac{3}{4}(20) = 15$ s. $t_2 = \frac{1}{4}(20) = 5$ s.

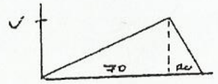
(b) $v = u + at \Rightarrow v = 0 + (1)(15) = 15$ m/s.

(c)



s = area under curve $= \frac{1}{2}(20)(15) = 150$ m.

7. $t_1 : t_2 = 7 : 2 = \frac{7}{9} : \frac{2}{9}$. $\therefore t_1 = \frac{7}{9}(90) = 70$ s and $t_2 = 20$ s.



$v = u + at = 0 + (2)(70) = 140$ m/s.

s = area $= \frac{1}{2}(90)(140) = 6300$ m = 6.3 km.

8. $s = ut + \frac{1}{2}at^2 \Rightarrow 18 = u(2) + \frac{1}{2}(a)(4) \Rightarrow u + a = 9.$ (1st part)

$s = ut + \frac{1}{2}at^2 \Rightarrow 48 = u(4) + \frac{1}{2}(a)(16) \Rightarrow u + 2a = 12.$ (1st and 2nd parts)

Solving these gives (i) $a = 3$ m/s^2 (ii) $u = 6$ m/s.

(iii) First 6 seconds: $s = ut + \frac{1}{2}at^2 \Rightarrow s = (6)(6) + \frac{1}{2}(3)(36) = 90$ m

The distance travelled = 90 − 48 = 42 m.

9. 1st part: $s = ut + \frac{1}{2}at^2 \Rightarrow 39 = u(1) + \frac{1}{2}a(1)^2 \Rightarrow 2u + a = 78.$

1st and 2nd parts: $76 = u(2) + \frac{1}{2}a(2)^2 \Rightarrow 2u + 2a = 76.$

First three parts: $111 = u(3) + \frac{1}{2}a(3)^2 \Rightarrow 6u + 9a = 222 \Rightarrow u + 3a = 74.$

Solving the first two equations gives $a = -2$, $u = 40$.

Solving the last two equations gives $a = -2$, $u = 40$. $\therefore$ They are consistent.

Stopping: $v^2 = u^2 + 2as \Rightarrow 0 = (40)^2 + 2(-2)s \Rightarrow s = 400$ m.

$\therefore$ It will travel a further 400 − 111 = 289 m.

10. a to b: $s = ut + \frac{1}{2}at^2 \Rightarrow 20 = u(5) + \frac{1}{2}(a)(5)^2 \Rightarrow 2u + 5a = 8.$

a to c: $40 = u(8) + \frac{1}{2}a(8)^2 \Rightarrow 2u + 8a = 10.$

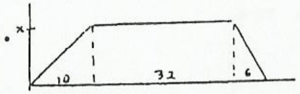
Solving these gives $u = \frac{7}{3}$ m/s , $a = \frac{2}{3}$ m/s^2.

a to d: $s = ut + \frac{1}{2}at^2 \Rightarrow 60 = \frac{7}{3}t + \frac{1}{2}(\frac{2}{3})t^2 \Rightarrow t^2 + 7t - 180 = 0.$

$\Rightarrow t = 10.4$ s (− 17.4 is rejected).

The time taken = 10.4 − 8 = 2.4 s.

11.

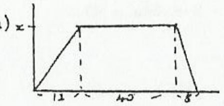


Area $= \frac{1}{2}(10)x + 32x + \frac{1}{2}(6)x = 1000$

$\Rightarrow x = 25$ m/s.

Distances are: $\frac{1}{2}(10)(25) = 125$; $32(25) = 800$; $\frac{1}{2}(6)(25) = 75$ m.

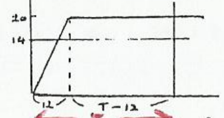
12. (a)



Area $= \frac{1}{2}(12)x + 40x + \frac{1}{2}(8)x = 1000$

$\Rightarrow \quad x = 20$ m/s.

(b)



$\text{Area}_1 = \text{Area}_2$

$\Rightarrow \quad 14T = \frac{1}{2}(12)(20) + 20(T - 12)$

$\Rightarrow \quad T = 20$ s $\quad\Rightarrow\quad s = 14T = 280$ m.

13. 1st part: $s = ut + \frac{1}{2}at^2 \quad\Rightarrow\quad 24 = u(2) + \frac{1}{2}a(2)^2 \quad\Rightarrow\quad u + a = 12.$

1st and 2nd parts: $48 = u(3) + \frac{1}{2}a(3)^2 \quad\Rightarrow\quad 2u + 3a = 32.$

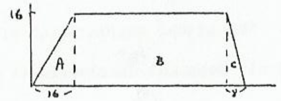
Solving these gives $a = 8,\ u = 4.$

First three parts: $72 = 4t + \frac{1}{2}(8)t^2 \quad\Rightarrow\quad t^2 + t - 18 = 0$

$\Rightarrow \quad t = 3.772 \qquad (t = -4.772$ is rejected).

Time taken $= 3.772 - 3 = 0.772$ seconds.

14. (i)



Distance in A $= \frac{1}{2}(16)(16) = 128$ m.

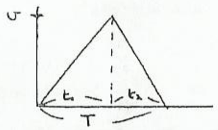
Distance in C $= \frac{1}{2}(8)(16) = 64$ m.

Total distance travelled $= 192$ m.

Remainder for B $= 300 - 192 = 108$ m. Time taken $= \frac{108}{16} = 6.75$ s.

Total time $= 16 + 6.75 + 8 = 30.75$ seconds.

(ii)



Let T = the time taken.

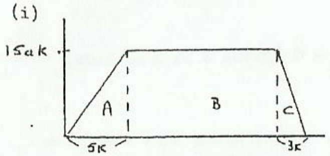
$t_1 : t_2 = d : a = 2 : 1 = \frac{2}{3} : \frac{1}{3}.$

$\therefore t_1 = \frac{2}{3}T$ and $t_2 = \frac{1}{3}T.$

$v = u + at \quad\Rightarrow\quad v = 0 + (1)(\frac{2}{3}T) = \frac{2}{3}T.$

Area $= 300 \quad\Rightarrow\quad \frac{1}{2}(T)(\frac{2}{3}T) = 300 \quad\Rightarrow\quad T = 30$ seconds.

15. (i)



Distance in A $= \frac{1}{2}(5k)(15ak) = 37\frac{1}{2}ak^2.$

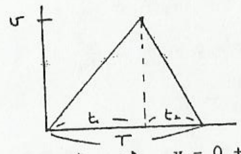
Distance in C $= \frac{1}{2}(3k)(15ak) = 22\frac{1}{2}ak^2.$

Remainder $= 90ak^2 - 37\frac{1}{2}ak^2 - 22\frac{1}{2}ak^2 = 30ak^2.$

Time for B $= \frac{30ak^2}{15ak} = 2k$ seconds.

Total time $= 5k + 2k + 3k = 10k$ seconds.

(ii)



Let T = the total time.

$t_1 : t_2 = d : a = 5a : 3a = \frac{5}{8} : \frac{3}{8}.$

$\therefore t_1 = \frac{5}{8}T$ and $t_2 = \frac{3}{8}T.$

$v = u + at \quad\Rightarrow\quad v = 0 + (3a)(\frac{5}{8}T) = \frac{15}{8}aT.$

Area $= 90ak^2 \quad\Rightarrow\quad \frac{1}{2}(T)(\frac{15}{8}aT) = 90ak^2 \quad\Rightarrow\quad T^2 = 96k^2 \quad\Rightarrow\quad T = 4\sqrt{6}k.$

16. $s_1 = 0 + \frac{1}{2}(\frac{1}{2})t^2 = \frac{1}{4}t^2. \qquad s_2 = 0 + \frac{1}{2}(1)t^2 = \frac{1}{2}t^2.$

Distance apart, $s = s_1 + s_2 = \frac{3}{4}t^2.$

When $t = 10$, $s = \frac{3}{4}(100) = 75$ m.

When $s = 108$, $\frac{3}{4}t^2 = 108 \quad\Rightarrow\quad t^2 = 144 \quad\Rightarrow\quad t = 12$ s.

This is $12 - 10 = 2$ seconds later.

17. $s_1 = 10t + \frac{1}{2}t^2 \ ; \qquad s_2 = 20t - 2\frac{1}{2}t^2. \qquad$ (Also $v_2 = 20 - 5t$).

Distance apart, $s = s_1 + s_2 = 30t - 2t^2.$

2nd one stops when $v_2 = 0 \quad\Rightarrow\quad 20 - 5t = 0 \quad\Rightarrow\quad t = 4$ s.

At $t = 4$, $s = 30(4) - 2(4)^2 = 88$ m.

When $s = 44$, $44 = 30t - 2t^2 \quad\Rightarrow\quad t^2 - 15t + 22 = 0 \quad\Rightarrow\quad t = 1.648$ s.

($t = 13.352$ is later and so it is rejected.)

18. $\left.\begin{array}{l} s = d \\ t = n \\ u = 0 \\ a = a \end{array}\right\} \Rightarrow \begin{array}{l} s = ut + \frac{1}{2}at^2 \\ d = \frac{1}{2}an^2 \quad \ldots A \end{array} \qquad \left.\begin{array}{l} s = d + k \\ t = 2n \\ u = 0 \\ a = a \end{array}\right\} \begin{array}{l} s = ut + \frac{1}{2}at^2 \\ \Rightarrow d + k = \frac{1}{2}a(2n)^2 \\ \Rightarrow d + k = 2an^2 \quad \ldots B \end{array}$

$4 \times$ Equation A $\quad\Rightarrow\quad 2an^2 = 4d.$ But $2an^2 = d + k.$

$\therefore \quad d + k = 4d \quad\Rightarrow\quad k = 3d \quad$ Q.E.D.

19.

After t seconds, Alberto has travelled $s_1 = 12t + \frac{1}{2}t^2.$

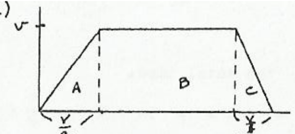
After t seconds, Gustav has travelled $s_2 = t^2.$

At both P_1 and P_2, $s_1 = s_2 + 22 \quad\Rightarrow\quad 12t + \frac{1}{2}t^2 = t^2 + 22$

$\Rightarrow \quad t^2 - 24t + 44 = 0 \quad\Rightarrow\quad t = 2,\ 22.$

Answer: (i) After 2 seconds $\qquad$ (ii) After 20 seconds <u>more</u> .

20. (i)



Distance A $= \frac{1}{2}(\frac{v}{a})v = \frac{v^2}{2a}$.

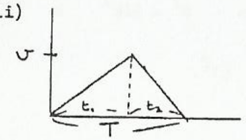
Distance C $= \frac{1}{2}(\frac{v}{b})v = \frac{v^2}{2b}$.

Remainder $= s - \frac{v^2}{2a} - \frac{v^2}{2b}$

Time taken in B $= (s - \frac{v^2}{2a} - \frac{v^2}{2b}) \div v = \frac{s}{v} - \frac{v}{2a} - \frac{v}{2b}$.

Total time taken $= \frac{v}{a} + (\frac{s}{v} - \frac{v}{2a} - \frac{v}{2b}) + \frac{v}{b} = \frac{v}{2a} + \frac{v}{2b} + \frac{s}{v}$. Q.E.D.

(ii)



Let T = the total time.

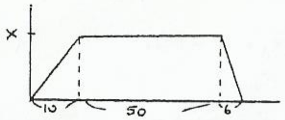
$t_1 : t_2 = b : a = \frac{b}{a+b} : \frac{a}{a+b}$.

$\therefore t_1 = \frac{bT}{a+b}$, $t_2 = \frac{aT}{a+b}$.

$v = u + at \Rightarrow v = 0 + (a)(\frac{bT}{a+b}) = \frac{abT}{a+b}$.

Area = s $\Rightarrow \frac{1}{2}(T)(\frac{abT}{a+b}) = s \Rightarrow T = \sqrt{2s(\frac{a+b}{ab})}$. Q.E.D.

21.

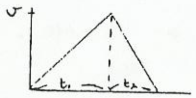


Area under the curve = 696

$\Rightarrow \frac{1}{2}(10)x + 50x + \frac{1}{2}(6)x = 696$

$\Rightarrow x = 12$.

$v = u + at \Rightarrow 12 = 0 + a_1(10) \Rightarrow a_1 = 1.2$ m/s^2. Similarily, $a_2 = 2$ m/s^2.



$t_1 : t_2 = 2 : 1.2 = 5 : 3 = \frac{5}{8} : \frac{3}{8}$.

$\therefore t_1 = \frac{5}{8}T$, $t_2 = \frac{3}{8}T$.

$v = u + at \Rightarrow v = 0 + (1.2)(\frac{5}{8}T) = \frac{3}{4}T$.

Area = 696 $\Rightarrow \frac{1}{2}(T)(\frac{3}{4}T) = 696 \Rightarrow T^2 = 1856 \Rightarrow T = 8\sqrt{29}$.

22. (a)
$$\left.\begin{aligned} t &= n - 1 \\ u &= u \\ a &= a \\ s &= s_1 \end{aligned}\right\} \quad s = ut + \tfrac{1}{2}at^2 \Rightarrow s_1 = u(n-1) + \tfrac{1}{2}a(n-1)^2 = un - u + \tfrac{1}{2}an^2 - an + \tfrac{1}{2}a.$$

$$\left.\begin{aligned} t &= n \\ u &= u \\ a &= a \\ s &= s_2 \end{aligned}\right\} \quad s = ut + \tfrac{1}{2}at^2 \Rightarrow s_2 = un + \tfrac{1}{2}an^2.$$

Distance travelled, $s = s_2 - s_1 = u + an - \frac{1}{2}a$. Q.E.D.

(b) When $n = 2$, $s = 17 \Rightarrow 17 = u + 2a - \frac{1}{2}a \Rightarrow u + 1\frac{1}{2}a = 17$.

When $n = 7$, $s = 47 \Rightarrow 47 = u + 7a - \frac{1}{2}a \Rightarrow u + 6\frac{1}{2}a = 47$.

Solving these gives $a = 6$, $u = 8$.

(i) $n = 10 \Rightarrow s = u + an - \frac{1}{2}a = 8 + (6)(10) - \frac{1}{2}(6) = 65$ m.

(ii) $n = n \Rightarrow s = 8 + 6n - \frac{1}{2}(6) = (6n + 5)$ m.

(iii) $s_n + s_{n+1} = 256 \Rightarrow 6n + 5 + 6(n+1) + 5 = 256 \Rightarrow n = 20$.

Answer: In the 20th and 21st seconds.

23. Take speeds, accelerations, distances relative to the goods train.

Let p = the passenger train and g = the goods train.

The initial relative speed, $u_{pg} = u_p - u_g = 80 - 30 = 50$ m/s.

The relative distance = 1500 m.

The final relative speed is zero, since the two trains must eventually be travelling at the same speed to avoid a crash.

$v^2 = u^2 + 2as \Rightarrow 0 = (50)^2 + 2a(1500) \Rightarrow a = -\frac{5}{6}$ m/s^2.

The relative deceleration is, therefore, $\frac{5}{6}$ m/s^2. The actual deceleration of the passenger train is $\frac{5}{6}$ m/s^2, since the goods train does not decelerate at all.

24. (a) Initial relative speed, $u = 20 - 8 = 12$ m/s.

Relative distance = 120 m. Final relative speed = 0 m/s.

$v^2 = u^2 + 2as \Rightarrow 0 = (12)^2 + 2a(120) \Rightarrow a = -\frac{3}{5}$ m/s^2.

(b) (i)
$$\left.\begin{aligned} u &= 12 \\ a &= -1 \\ s &= 120 - 66 = 54 \\ t &= ? \end{aligned}\right\} \quad \begin{aligned} & s = ut + \tfrac{1}{2}at^2 \\ \Rightarrow\ & 54 = 12t + \tfrac{1}{2}(-1)t^2 \Rightarrow t^2 - 24t + 54 = 0 \\ \Rightarrow\ & (t-6)(t-18) = 0 \Rightarrow t = 6, 18. \end{aligned}$$

Answer : After 6 seconds.

(ii)
$$\left.\begin{aligned} u &= 12 \\ a &= -1 \\ s &= s \\ t &= t \end{aligned}\right\} \quad \begin{aligned} & s = ut + \tfrac{1}{2}at^2 \\ \Rightarrow\ & s = 12t - \tfrac{1}{2}t^2 \\ \Rightarrow\ & \frac{ds}{dt} = 12 - t = 0 \text{ (Since s is a minimum)} \\ \Rightarrow\ & t = 12. \end{aligned}$$

At $t = 12$, $s = 12(12) - \frac{1}{2}(12)^2 = 72$ m. This means that they have travelled a distance of 72 m towards each other, and so the distance between them will be $120 - 72 = 48$ m.

Exercise 3.A

1.(i) $V_y = 0 \Rightarrow 28 - 9.8t = 0 \Rightarrow t = \frac{20}{7}$ s.

At $t = \frac{20}{7}$, $S_y = 28t - 4.9t^2 = 28(\frac{20}{7}) - 4.9(\frac{400}{49}) = 80 - 40 = 40$ m.

(ii) $S_y = 0 \Rightarrow 28t - 4.9t^2 = 0 \Rightarrow t = 0$ or $t = \frac{40}{7}$.

At $t = \frac{40}{7}$, $S_x = 21t = 21(\frac{40}{7}) = 120$ m.

2. (i) $S_y = 21t - 4.9t^2 = 22.4 \Rightarrow 7t^2 - 30t + 32 = 0$

$\Rightarrow (7t - 16)(t - 2) = 0 \Rightarrow t = \frac{16}{7}$ or $t = 2$.

(ii) When it is at its greatest height, $V_y = 0$ and $V_x = 14$, as always.

$\therefore \vec{v} = 14\vec{i} + 0\vec{j}$.

3. (i) $V_x = 70$. At $t = 10$, $V_y = 105 - 9.8t = 105 - 98 = 7$. $\therefore \vec{v} = 70\vec{i} + 7\vec{j}$.

$v = \sqrt{4900 + 49} = \sqrt{4949} = 70.35$ m/s.

$\tan\theta = \frac{7}{70} = 0.1 = \theta = 5^\circ 43'$. Direction: E $5^\circ 43'$ N.

(ii) $S_y = 0 \Rightarrow 105t - 4.9t^2 = 0 \Rightarrow t = 0$ or $t = \frac{105}{4.9} = \frac{150}{7}$.

At $t = \frac{150}{7}$, $S_x = 70t = 70(\frac{150}{7}) = 1500$ m.

4. (i) $V_y = 0 \Rightarrow 35 - 9.8t = 0 \Rightarrow t = \frac{35}{9.8} = \frac{25}{7}$.

(ii) $V_y = 10.5 \Rightarrow 35 - 9.8t = 10.5 \Rightarrow t = \frac{24.5}{9.8} = \frac{5}{2}$.

(iii) $V_y = -10.5 \Rightarrow 35 - 9.8t = -10.5 \Rightarrow t = \frac{45.5}{9.8} = \frac{65}{14}$.

Midway between (ii) and (iii) is $\frac{5/2 + 65/14}{2} = \frac{25}{7} =$ (i). Q.E.D.

5. $S_y = -490 \Rightarrow -4.9t^2 = -490 \Rightarrow t = 10$.

At $t = 10$, $S_x = 200t = 200(10) = 2000$ m.

6. $\vec{u} = 30\vec{i} + 40\vec{j}$ $\therefore \vec{r} = 30t\,\vec{i} + (40t - 4.9t^2)\vec{j}$, $\vec{v} = 30\vec{i} + (40 - 9.8t)\vec{j}$.

At $t = 1$, $\vec{v} = 30\vec{i} + 30.2\vec{j} \Rightarrow |\vec{v}| = \sqrt{30^2 + (30.2)^2} = 43$ m/s.

$\tan\theta = \frac{30.2}{30} \Rightarrow \theta = 45^\circ$. Direction: E 45° N.

$S_y = 0 \Rightarrow 40t - 4.9t^2 = 0 \Rightarrow t = 0$ or $t = \frac{40}{4.9} = \frac{400}{49}$.

At $t = \frac{400}{49}$, $S_x = 30t = 30(\frac{400}{49}) = \frac{12,000}{49} = 245$ m.

7. (i) $V_y = 0 \Rightarrow 98 - 9.8t = 0 \Rightarrow t = 10$.

(ii) At $t = 10$, $S_x = 10t = 10(10) = 100$ m, $S_y = 98t - 4.9t^2 = 98(10) - 4.9(100) = 490$ m.

$\therefore \vec{r} = 100\vec{i} + 490\vec{j}$.

8. (i) $\cos\alpha = \frac{2}{\sqrt{5}}$, $\sin\alpha = \frac{1}{\sqrt{5}}$

$\vec{u} = 7\sqrt{5}\cos\alpha\,\vec{i} + 7\sqrt{5}\sin\alpha\,\vec{j} = 14\vec{i} + 7\vec{j}$.

(ii) $S_y = 0 \Rightarrow 7t - 4.9t^2 = 0 \Rightarrow t = 0$ or $t = \frac{7}{4.9} = \frac{10}{7}$.

At $t = \frac{10}{7}$, $S_x = 14t = 14(\frac{10}{7}) = 20$ m.

9. $\vec{u} = 35\cos\theta\,\vec{i} + 35\sin\theta\,\vec{j} = 35(\frac{4}{5})\vec{i} + 35(\frac{3}{5})\vec{j} = 28\vec{i} + 21\vec{j}$.

$S_y = 10 \Rightarrow 21t - 4.9t^2 = 10 \Rightarrow 49t^2 - 210t + 100 = 0$

$\Rightarrow t = 0.546$ (or 3.740).

At $t = 0.546$, $V_x = 28$ and $V_y = 21 - 9.8t = 21 - 9.8(0.546) = 15.65$.

$\therefore \vec{v} = 28\,\vec{i} + 15.65\,\vec{j} \Rightarrow |\vec{v}| = \sqrt{28^2 + (15.65)^2} = 32.08$ m/s.

10. $S_y = 0 \Rightarrow 7t - 4.9t^2 = 0 \Rightarrow t = 0$ or $t = \frac{7}{4.9} = \frac{10}{7}$.

At $t = \frac{10}{7}$, $S_x = 10t = 10(\frac{10}{7}) = \frac{100}{7}$ m $= R$, the range.

$\frac{3}{4}R = \frac{75}{7}$ m. $S_x = \frac{75}{7} \Rightarrow t = \frac{75}{70} = \frac{15}{14}$.

At $t = \frac{15}{14}$, $S_y = 7(\frac{15}{14}) - 4.9(\frac{225}{196}) = 7.5 - 5.625 = 1.875$ m.

11. $S_y = 0 \Rightarrow 4t - 4.9t^2 = 0 \Rightarrow t = 0$ or $t = \frac{4}{4.9} = \frac{40}{49}$.

At $t = \frac{40}{49}$, $S_x = 3t = 3(\frac{40}{49}) = \frac{120}{49} = R$, the range. $\therefore \frac{1}{5}R = \frac{24}{49}$ m.

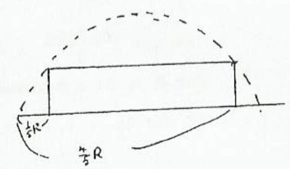
When is $S_x = \frac{24}{49}$? When $3t = \frac{24}{49} \Rightarrow t = \frac{8}{49}$.

At $t = \frac{8}{49}$, $S_y = 4(\frac{8}{49}) - 4.9(\frac{64}{2401}) = \frac{32}{49} - \frac{64}{490} = \frac{128}{245}$.

By symmetry, it will reach the same height when

$S_x = \frac{4}{5}R$. The time will be $\frac{4}{5}$ of the time of

flight $= \frac{4}{5}(\frac{40}{49}) = \frac{32}{49}$ s.



12. (a) $(7t + 50)(t - 10)$

(b) $S_y = -350 \Rightarrow 14t - 4.9t^2 = -350 \Rightarrow 7t^2 - 20t - 50 = 0$

$\Rightarrow (7t + 50)(t - 10) = 0 \Rightarrow t = 10$ ($t = -50/7$ is rejected).

At $t = 10$, $S_x = 10t = 10(10) = 100$ m.

13. $S_y = 49t - 4.9t^2 = 0 \Rightarrow t = 0$ or $t = 10$.

At $t = 10$, $S_x = 50t = 50(10) = 500$ m.

14. To find the maximum height, H: $V_y = 0 \Rightarrow 14 - 9.8t = 0 \Rightarrow t = \frac{14}{9.8} = \frac{10}{7}$.

At $t = \frac{10}{7}$, $S_y = 14t - 9.8t^2 = 14(\frac{10}{7}) - 4.9(\frac{100}{49}) = 20 - 10 = 10$ m $= H$.

$\frac{3}{4}H = \frac{3}{4}(10) = 7.5$ m.

$S_y = 7.5 \Rightarrow 14t - 4.9t^2 = 7.5 \Rightarrow 49t^2 - 140t + 75 = 0$

$\Rightarrow (7t - 5)(7t - 15) = 0 \Rightarrow t = \frac{5}{7}$ or $\frac{15}{7}$.

15. $S_y = -82.5 \Rightarrow 8t - 4.9t^2 = -82.5 \Rightarrow 49t^2 - 80t - 825 = 0$

$\Rightarrow t = 5$ or $-330/98$.

At $t = 5$, $S_x = 12t = 12(5) = 60$ m.

16. (i) $S_y = -78.4 \Rightarrow -4.9t^2 = -78.4 \Rightarrow t^2 = 16 \Rightarrow t = 4$.

(ii) $\vec{u} = 98 \text{ Cos } 30^\circ \vec{i} + 98 \text{ Sin } 30^\circ \vec{j} = 84.868\,\vec{i} + 49\,\vec{j}$.

$S_y = -78.4 \Rightarrow 49t - 4.9t^2 = -78.4 \Rightarrow 49t^2 - 490t - 784 = 0$

$\Rightarrow t^2 - 10t - 16 = 0 \Rightarrow t = 11.4$ (-1.4 is rejected).

17. $S_x = 12t$, $S_y = kt - 4.9t^2$.

$S_x = 30 \Rightarrow 12t = 30 \Rightarrow t = 2.5$.

At $t = 2.5$, $S_y = 9.375 \Rightarrow k(2.5) - 4.9(2.5)^2 = 9.375 \Rightarrow k = 16$.

18. (i) $\vec{u} = u \text{ Cos}\alpha\, \vec{i} + u \text{ Sin}\alpha\, \vec{j}$.

(ii) $V_y = 0 \Rightarrow u \text{ Sin}\alpha - gt = 0 \Rightarrow t = \frac{u \text{ Sin}\alpha}{g}$.

At $t = \frac{u \text{ Sin}\alpha}{g}$, $S_y = u \text{ Sin}\alpha\, t - \frac{1}{2}gt^2 = u \text{ Sin}\alpha\,(\frac{u \text{ Sin}\alpha}{g}) - \frac{1}{2}g(\frac{u^2\text{Sin}^2\alpha}{g^2})$
$= \frac{u^2\text{Sin}^2\alpha}{2g}$.

(iii) $S_y = 0 \Rightarrow u \text{ Sin}\alpha\, t - \frac{1}{2}gt^2 = 0 \Rightarrow t = 0$ or $\frac{2u \text{ Sin}\alpha}{g}$.

At $t = \frac{2u \text{ Sin}\alpha}{g}$, $S_x = u \text{ Cos}\alpha\, t = u \text{ Cos}\alpha\,(\frac{2u \text{ Sin}\alpha}{g}) = \frac{u^2 \text{ Sin } 2\alpha}{g} = R$.

For R to be a maximum, $\text{Sin } 2\alpha = 1 \Rightarrow R = u^2/g$.

If $\text{Sin } 2\alpha = 1$, then $2\alpha = 90^\circ \Rightarrow \alpha = 45^\circ$.

19. From q.18, $\frac{u^2 \text{ Sin}^2\alpha}{2g} = \frac{2u^2 \text{ Sin}\alpha \text{ Cos}\alpha}{g} \Rightarrow \text{Tan}\alpha = 4 \Rightarrow \alpha = 76^\circ$.

20. $\frac{u^2 \text{ Sin}^2\alpha}{2g} = 3.6$ ....1 $\qquad \frac{2u^2 \text{ Sin}\alpha \text{ Cos}\alpha}{g} = 19.2$ 2

Dividing 1 by 2 $\Rightarrow \frac{\text{Tan}\,\alpha}{4} = \frac{3}{16} \Rightarrow \text{Tan}\alpha = \frac{3}{4} \Rightarrow \text{Sin}\alpha = \frac{3}{5}$.

Putting this result into 1 gives

$\frac{u^2(9/25)}{19.6} = 3.6 \Rightarrow u^2 = 196 \Rightarrow u = 14$ m/s.

$R_{max} = u^2/g = 196/9.8 = 20$ m.

21. (a) 30°, 150°.

(b) $R = \frac{u^2 \text{ Sin } 2\alpha}{g} = 40 \Rightarrow \frac{784 \text{ Sin } 2\alpha}{9.8} = 40 \Rightarrow \text{Sin } 2\alpha = \frac{1}{2}$

$\Rightarrow 2\alpha = 30^\circ$ or $150^\circ \Rightarrow \alpha = 15^\circ$ or 75°.

22. $H = \frac{u^2 \text{ Sin}^2\alpha}{2g} \Rightarrow 2.5 = \frac{100 \text{ Sin}^2\alpha}{19.6} \Rightarrow \text{Sin}^2\alpha = 0.49 \Rightarrow \text{Sin}\alpha = 0.7$

23. As in q. 18(iii), $\alpha = 45^\circ$.

$H = \frac{u^2 \text{ Sin}^2 45^\circ}{2g} = \frac{u^2(\frac{1}{2})}{2g} = \frac{u^2}{4g}$. $\qquad R = \frac{u^2 \text{Sin } 2(45^\circ)}{g} = \frac{u^2}{g}$.

Therefore, H : R = 1 : 4.

24. (a) $\text{Sin}^2A + \text{Cos}^2A = 1$. $\div \text{Cos}^2A \Rightarrow \text{Tan}^2A + 1 = \text{Sec}^2A$.

(b) $u = 35\sqrt{5} \text{ Cos}\alpha\, \vec{i} + 35\sqrt{5} \text{ Sin}\alpha\, \vec{j}$.

$S_x = 35\sqrt{5} \text{ Cos}\alpha\, t = 350 \Rightarrow t = \frac{350}{35\sqrt{5} \text{ Cos}\alpha} = \frac{10}{\sqrt{5} \text{ Cos}\alpha}$

At $t = \frac{10}{\sqrt{5} \text{ Cos}\alpha}$, $S_y = 35\sqrt{5} \text{ Sin}\alpha\, t - 4.9t^2 = 210$.

$\Rightarrow 35\sqrt{5} \text{ Sin}\alpha\,(\frac{10}{\sqrt{5} \text{ Cos}\alpha}) - 4.9\,(\frac{100}{5 \text{ Cos}^2\alpha}) = 210$.

$\Rightarrow 350 \text{ Tan}\,\alpha - 98 \text{ Sec}^2\alpha = 210 \Rightarrow 25 \text{ Tan}\alpha - 7 \text{ Sec}^2\alpha = 15$

$\Rightarrow 25 \text{ Tan}\alpha - 7(\text{Tan}^2\alpha + 1) = 15 \Rightarrow 25T - 7T^2 - 7 = 15$ (where $T = \text{Tan}\alpha$)

$\Rightarrow 7T^2 - 25T + 22 = 0 \Rightarrow (7T - 11)(T - 2) = 0$

$\Rightarrow \text{Tan}\alpha = \frac{11}{7}$ or 2. $\left(\Rightarrow \text{Cos}\alpha = \frac{7}{\sqrt{170}} \text{ or } \frac{1}{\sqrt{5}}\right)$

Now $t = \frac{10}{\sqrt{5} \text{ Cos}\alpha} = \frac{10}{7}\sqrt{34}$ secs or 10 secs.

Exercise 3.B

Answers in text ; no solutions necessary.

Exercise 3.C

1.(i) $S_y = u\sin\alpha\, t - \frac{1}{2}g\cos\beta\, t^2 = 0$

$\Rightarrow \quad t = 0 \quad \text{or} \quad t = 2u\sin\alpha / g\cos\beta \quad = \quad$ time of flight.

(ii) $V_y = 0 \quad \Rightarrow \quad u\sin\alpha - g\cos\beta\, t = 0$

$\Rightarrow \quad t = u\sin\alpha / g\cos\beta \quad = \quad \frac{1}{2}$(time of flight).

2. $S_y = 0 \quad \Rightarrow \quad u\sin\alpha\, t - \frac{1}{2}g\cos\beta\, t^2 = 0$

$\Rightarrow \quad t = 0 \quad \text{or} \quad t = 2u\sin\alpha / g\cos\beta \quad = \quad$ time of flight.

At $t = 2u\sin\alpha / g\cos\beta$, $\quad S_x = u\cos\alpha\, t + \frac{1}{2}g\sin\beta\, t^2$

$$= u\cos\alpha\left(\frac{2u\sin\alpha}{g\cos\beta}\right) + \frac{1}{2}g\sin\beta\left(\frac{4u^2\sin^2\alpha}{g^2\cos^2\beta}\right)$$

$$= \frac{2u^2\cos\alpha\sin\alpha\cos\beta + 2u^2\sin\beta\sin^2\alpha}{g\cos^2\beta}$$

$$= \frac{2u^2\sin\alpha(\cos\alpha\cos\beta + \sin\beta\cdot\sin\alpha)}{g\cos^2\beta} = \frac{2u^2\sin\alpha\cos(\alpha-\beta)}{g\cos^2\beta}$$

$$= \frac{u^2}{g\cos^2\beta}(\sin(2\alpha-\beta) + \sin\beta) \quad = \quad \text{the range.}$$

This range is a maximum if $\sin(2\alpha - \beta) = 1$.

$\therefore$ The maximum range $= \frac{u^2}{g\cos^2\beta}(1 + \sin\beta)$

$$= \frac{u^2(1+\sin\beta)}{g(1-\sin^2\beta)} = \frac{u^2}{g(1-\sin\beta)}. \qquad \text{Q.E.D.}$$

Also, $\sin(2\alpha - \beta) = 1 \quad \Rightarrow \quad 2\alpha - \beta = 90^\circ \quad \Rightarrow \quad \alpha = \frac{\beta + 90^\circ}{2}$ Q.E.D.

3. $S_y = 0 \quad \Rightarrow \quad u\sin 45^\circ\, t - \frac{1}{2}g\cos 30^\circ\, t^2 = 0 \quad \Rightarrow \quad \frac{u}{\sqrt{2}}t - \frac{\sqrt{3}}{4}gt^2 = 0$

$\Rightarrow \quad t = 0 \quad \text{or} \quad t = \frac{4u}{\sqrt{6}g} \quad = \quad$ time of flight.

At $t = \frac{4u}{\sqrt{6}g}$, $\quad S_x = u\cos 45^\circ\, t - \frac{1}{2}g\sin 30^\circ\, t^2$

$$= u\left(\frac{1}{\sqrt{2}}\right)\left(\frac{4u}{\sqrt{6}g}\right) - \frac{1}{2}g\left(\frac{1}{2}\right)\left(\frac{16u^2}{6g^2}\right)$$

$$= \frac{2u^2}{\sqrt{3}g} - \frac{2u^2}{3g} = \frac{2u^2}{3g}(\sqrt{3} - 1)$$

$V_y = 0 \quad \Rightarrow \quad u\sin 45^\circ - g\cos 30^\circ\, t = 0 \quad \Rightarrow \quad \frac{u}{\sqrt{2}} - \frac{\sqrt{3}g}{2}t = 0 \quad \Rightarrow \quad t = \frac{2u}{\sqrt{6}g}$.

At $t = \frac{2u}{\sqrt{6}g}$, $\quad S_y = \frac{u}{\sqrt{2}}t - \frac{\sqrt{3}}{4}gt^2 = \frac{u}{\sqrt{2}}\left(\frac{2u}{\sqrt{6}g}\right) - \frac{\sqrt{3}}{4}g\left(\frac{4u^2}{6g^2}\right)$

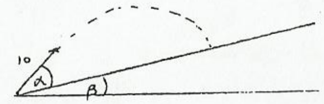
$$= \frac{u^2}{\sqrt{3}g} - \frac{\sqrt{3}u^2}{6g} = \frac{2\sqrt{3}u^2 - \sqrt{3}u^2}{6g} = \frac{\sqrt{3}u^2}{6g}.$$

4. $\cos\alpha = 4/5$

$\sin\alpha = 3/5$

$\cos\beta = 12/13$

$\sin\beta = 5/13$



$S_y = 0 \quad \Rightarrow \quad 10\sin\alpha\, t - \frac{1}{2}g\cos\beta\, t^2 = 0 \quad \Rightarrow \quad 6t - \frac{6}{13}gt^2 = 0$

$\Rightarrow \quad t = 0 \quad \text{or} \quad t = 13/g \quad = \quad$ time of flight.

At $t = 13/g$, $\quad S_x = 10\cos\alpha\, t - \frac{1}{2}g\sin\beta\, t^2$

$$= 10\left(\frac{4}{5}\right)\left(\frac{13}{g}\right) - \frac{1}{2}g\left(\frac{5}{13}\right)\left(\frac{169}{g^2}\right) = \frac{143}{2g} = R, \text{ the range.}$$

2/5ths of the time of flight $= \frac{2}{5} \times \frac{13}{g} = \frac{26}{5g}$.

At $t = 26/5g$, $\quad S_x = 10\left(\frac{4}{5}\right)\left(\frac{26}{5g}\right) - \frac{1}{2}g\left(\frac{5}{13}\right)\left(\frac{676}{25g^2}\right) = \frac{208}{5g} - \frac{26}{5g} = \frac{182}{5g}$

Now, $\frac{182}{5g} = \frac{28}{55}\left(\frac{143}{2g}\right) = \frac{28}{55}R$.

5. $S_y = 10\sin(\alpha - 30^\circ)t - \frac{1}{2}\cos 30^\circ\, gt^2$.

$S_x = 10\cos(\alpha - 30^\circ)t - \frac{1}{2}\sin 30^\circ\, gt^2$.

(i) If $\alpha = 75^\circ$, $\quad S_y = 10\sin 45^\circ\, t - \frac{1}{2}\cos 30^\circ\, gt^2 = 0$

$\Rightarrow \quad \frac{10}{\sqrt{2}}t - \frac{\sqrt{3}}{4}gt^2 = 0 \quad \Rightarrow \quad t = 0 \quad \text{or} \quad t = \frac{40}{\sqrt{6}g}$.

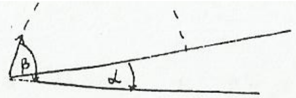
At $t = \frac{40}{\sqrt{6}g}$, $\quad S_x = 10\left(\frac{1}{\sqrt{2}}\right)\left(\frac{40}{\sqrt{6}g}\right) - \frac{1}{2}\left(\frac{1}{2}\right)g\left(\frac{1600}{6g}\right) = \frac{200}{\sqrt{3}g} - \frac{200}{3g} = \frac{200(\sqrt{3}-1)}{3g}$.

(ii) If $\alpha = 60^\circ$, $\quad S_y = 10\sin 30^\circ\, t - \frac{1}{2}\cos 30^\circ\, gt^2 = 0$

$\Rightarrow \quad \frac{10}{\sqrt{2}}t - \frac{\sqrt{3}}{4}gt^2 = 0 \quad \Rightarrow \quad t = 0 \quad \text{or} \quad t = \frac{20}{\sqrt{3}g}$.

At $t = \frac{20}{\sqrt{3}g}$, $\quad S_x = 10\left(\frac{\sqrt{3}}{2}\right)\left(\frac{20}{\sqrt{3}g}\right) - \frac{1}{2}\left(\frac{1}{2}\right)g\left(\frac{400}{3g}\right) = \frac{100}{g} - \frac{100}{3g} = \frac{200}{3g}$.

6. Tan 2 = Cos $\sqrt{5}$ = Sin $\sqrt{5}$. $\vec{u} = 7\vec{i} + 14\vec{j}$.



Taking the horizontal as the x-axis,

$V_y = 0 \Rightarrow 14 - \frac{1}{2}gt = 0 \Rightarrow t = \frac{28}{g}$ seconds = time to reach greatest height above the horizontal.

Now taking the line of greatest slope as the x-axis, $\vec{u} = u\text{Cos}(\beta - \alpha)\vec{i} + u\text{Sin}(\beta - \alpha)\vec{j}$.

But, $\text{Cos}(\beta - \alpha) = \text{Cos}\beta\,\text{Cos}\alpha + \text{Sin}\beta\,\text{Sin}\alpha = \frac{1}{\sqrt{5}}\cdot\frac{2}{\sqrt{5}} + \frac{2}{\sqrt{5}}\cdot\frac{1}{\sqrt{5}} = \frac{4}{5}$.

Therefore, $\text{Sin}(\beta - \alpha) = \frac{3}{5}$.

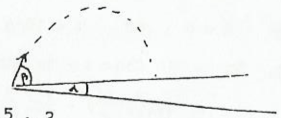
$V_y = 0 \Rightarrow u\text{Sin}(\beta - \alpha) - \frac{1}{2}g\,\text{Cos}\alpha\, t = 0$

$\Rightarrow 7\sqrt{5}(\frac{3}{5}) - \frac{1}{2}g(\frac{2}{\sqrt{5}})t = 0 \Rightarrow \frac{21\sqrt{5}}{5} - \frac{\sqrt{5}}{5}gt = 0 \Rightarrow t = \frac{21}{g}$ secs.

The ratio $t_1 : t_2 = \frac{28}{g} : \frac{21}{g} = 4 : 3$.

7. $\text{Sin}\,\alpha = 5/13$; $\text{Cos}\,\alpha = 12/13$.

$\text{Sin}\,\beta = 4/5$; $\text{Cos}\,\beta = 3/5$.



$S_x = u\text{Cos}\beta\; t - \frac{1}{2}g\text{Sin}\,\alpha\; t^2 = 10(\frac{3}{5})t - \frac{1}{2}g(\frac{5}{13})t^2 = 6t - \frac{5}{26}gt^2$.

$S_y = u\text{Sin}\,\beta\; t - \frac{1}{2}g\text{Cos}\,\alpha\; t^2 = 10(\frac{4}{5})t - \frac{1}{2}g(\frac{12}{13})t^2 = 8t - \frac{6}{13}gt^2$.

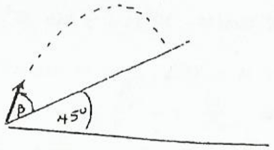
$S_x = 2S_y \Rightarrow 6t - \frac{5}{26}gt^2 = 2(8t - \frac{6}{13}gt^2)$

$\Rightarrow \frac{19}{26}gt^2 - 10t = 0 \Rightarrow t = 0$ or $t = \frac{260}{19g}$: Answer.

8. $\text{Sin}\,\beta = \frac{1}{\sqrt{5}}$; $\text{Cos}\,\beta = \frac{2}{\sqrt{5}}$

$S_y = 0 \Rightarrow u\text{Sin}\beta\, t - \frac{1}{2}g\text{Cos}\;45^\circ\; t^2 = 0$

$\Rightarrow 4t - \frac{1}{2\sqrt{2}}gt^2 = 0$



$\Rightarrow t = 0$ or $t = \frac{8\sqrt{2}}{g}$ = time of flight.

$V_x = u\text{Cos}\,\beta - g\text{Sin}\;45^\circ\; t = 8 - \frac{g}{\sqrt{2}}t$.

At $t = \frac{8\sqrt{2}}{g}$, $V_x = 8 - \frac{g}{\sqrt{2}}(\frac{8\sqrt{2}}{g}) = 8 - 8 = 0$ m/s.

$V_y = u\text{Sin}\,\beta - g\text{Cos}\;45^\circ\; t = 4 - \frac{g}{\sqrt{2}}t$.

At $t = \frac{8\sqrt{2}}{g}$, $V_y = 4 - \frac{g}{\sqrt{2}}(\frac{8\sqrt{2}}{g}) = 4 - 8 = -4$ m/s.

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{0 + 16} = 4$ m/s.

9. $\vec{u} = 60\vec{i} + 20\vec{j}$.

$V_y = 0 \Rightarrow 20 - \frac{1}{2}g(\frac{1}{\sqrt{2}})t = 0 \Rightarrow t = \frac{40\sqrt{2}}{g}$.

$V_x = 60 - \frac{1}{2}g(\frac{1}{\sqrt{2}})t = 60 - \frac{1}{2\sqrt{2}}gt$.

At $t = \frac{40\sqrt{2}}{g}$, $V_x = 60 - \frac{1}{2\sqrt{2}}g(\frac{40\sqrt{2}}{g}) = 60 - 40 = 20$ m/s.

Since $V_y = 0$, its speed is 20 m/s.

10 (i) $S_y = 0 \Rightarrow \sqrt{26}\,\text{Sin}\,\alpha\; t - \frac{1}{2}g(\frac{5}{\sqrt{26}})t^2 = 0 \Rightarrow t = 0$ or $t = \frac{52\text{Sin}\,\alpha}{5g}$.

(ii) $S_x = \sqrt{26}\,\text{Cos}\,\alpha\; t - \frac{1}{2}g(\frac{1}{\sqrt{26}})t^2$.

At $t = \frac{52\text{Sin}\,\alpha}{5g}$, $S_x = \sqrt{26}\,\text{Cos}\,\alpha(\frac{52\text{Sin}\,\alpha}{5g}) - \frac{1}{2}g(\frac{1}{\sqrt{26}})(\frac{2704\,\text{Sin}^2\alpha}{25g^2})$

$= \frac{52\sqrt{26}\text{Sin}\,\alpha\;\text{Cos}\,\alpha}{5g} - \frac{52\sqrt{26}\,\text{Sin}^2\alpha}{25g}$

$= \frac{52\sqrt{26}\,\text{Sin}\,\alpha}{25g}(5\text{Cos}\,\alpha - \text{Sin}\,\alpha)$ = the range.

(iii) If $\alpha = \beta$ then $\text{Sin}\,\alpha = \text{Sin}\,\beta = \frac{1}{\sqrt{26}}$; $\text{Cos}\,\alpha = \frac{5}{\sqrt{26}}$.

∴ The range $= \frac{52\sqrt{26}(\frac{1}{\sqrt{26}})}{25g}(\frac{25}{\sqrt{26}} - \frac{1}{\sqrt{26}}) = \frac{48\sqrt{26}}{25g}$.

(iv) If $\alpha = 2\beta$, then $\text{Sin}\,\alpha = \text{Sin}\;2\beta = 2\text{Sin}\beta\,\text{Cos}\beta = 2(\frac{1}{\sqrt{26}})(\frac{5}{\sqrt{26}}) = \frac{5}{13}$.

Also, $\text{Cos}\,\alpha = \text{Cos}\;2\beta = \text{Cos}^2\beta - \text{Sin}^2\beta = \frac{25}{26} - \frac{1}{26} = \frac{12}{13}$.

∴ The range $= \frac{52\sqrt{26}(\frac{5}{13})}{25g}(\frac{60}{13} - \frac{5}{13}) = \frac{44\sqrt{26}}{13g}$.

Exercise 3.D

1. $S_y = 0 \Rightarrow u\text{Sin}(\alpha - \beta)t - \frac{1}{2}g\text{Cos}\,\beta\; t^2 = 0$

$\Rightarrow t = 0$ or $t = \frac{2u\text{Sin}(\alpha - \beta)}{g\text{Cos}\,\beta}$.

Now, $V_x = u\text{Cos}(\alpha - \beta) - g\text{Sin}\,\beta\, t$. But $V_x = 0$ at $t = \frac{2u\,\text{Sin}(\alpha - \beta)}{g\text{Cos}\,\beta}$

∴ $u\,\text{Cos}(\alpha - \beta) - g\,\text{Sin}\,\beta(\frac{2u\,\text{Sin}(\alpha - \beta)}{g\text{Cos}\,\beta}) = 0$. Divide by $u\text{Cos}(\alpha - \beta)$.

$= 1 - 2\text{Tan}(\alpha - \beta)\,\text{Tan}\,\beta = 0 \Rightarrow 2\text{Tan}(\alpha - \beta)\,\text{Tan}\beta = 1$.

1. (contd)

If $(\alpha - \beta) = \frac{\pi}{4}$, then $\tan(\alpha - \beta) = 1$ and hence $2\,\mathrm{Tan}\,\beta = 1$

$\Rightarrow \mathrm{Tan}\,\beta = \frac{1}{2} \Rightarrow \mathrm{Sin}\,\beta = \frac{1}{\sqrt{5}}$ and $\mathrm{Cos}\,\beta = \frac{2}{\sqrt{5}}$

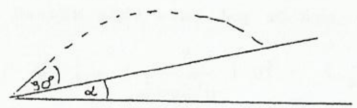
In this case $t = \dfrac{2u\,\mathrm{Sin}\,(\alpha - \beta)}{g\mathrm{Cos}\beta} \Rightarrow t = \dfrac{2u(1/\sqrt{2})}{g(2/\sqrt{5})} = \dfrac{\sqrt{5}u}{\sqrt{2}g}$

$Sx = u\mathrm{Cos}(\alpha - \beta)\,t - \frac{1}{2}g\,\mathrm{Sin}\,\beta\,t^2$.

At $t = \dfrac{\sqrt{5}u}{\sqrt{2}g}$, $Sx = u(\frac{1}{\sqrt{2}})(\frac{\sqrt{5}u}{\sqrt{2}g}) - \frac{1}{2}g(\frac{1}{\sqrt{5}})(\frac{5u^2}{2g^2}) = \dfrac{\sqrt{5}u^2}{2g} - \dfrac{\sqrt{5}u^2}{4g} = \dfrac{\sqrt{5}u^2}{4g}$ = Range.

2. $\mathrm{Cos}\,\alpha = \frac{2}{\sqrt{5}}$,

$\mathrm{Sin}\,\alpha = \frac{1}{\sqrt{5}}$.



$S_y = 0 \Rightarrow u\mathrm{Sin}30^\circ t - \frac{1}{2}g\,\mathrm{Cos}\,\alpha t^2 = 0 \Rightarrow \frac{1}{2}ut - \frac{1}{\sqrt{5}}gt^2 = 0$

$\Rightarrow t = 0$ or $t = \dfrac{\sqrt{5}u}{2g}$ = time of flight.

$V_y = u\mathrm{Sin}30^\circ - g\mathrm{Cos}\,\alpha t = \frac{1}{2}u - \frac{2}{\sqrt{5}}gt$

At $t = \dfrac{\sqrt{5}u}{2g}$, $V_y = \frac{1}{2}u - \frac{2}{\sqrt{5}}g(\frac{\sqrt{5}u}{2g}) = \frac{1}{2}u - u = -\frac{1}{2}u$

$Vx = u\mathrm{Cos}\,30^\circ - g\mathrm{Sin}\,\alpha t = \frac{\sqrt{3}}{2}u - \frac{1}{\sqrt{5}}gt$.

At $t = \dfrac{\sqrt{5}u}{2g}$, $Vx = \frac{\sqrt{3}}{2}u - \frac{1}{\sqrt{5}}g\,(\frac{\sqrt{5}u}{2g}) = \frac{\sqrt{3}}{2}u - \frac{1}{2}u = \left(\frac{\sqrt{3}-1}{2}\right)u$.

$\mathrm{Tan}\,\underline{l} = \dfrac{-Vy}{Vx} = \frac{1}{2}u \Big/ \dfrac{(\sqrt{3}-1)}{2}u = \dfrac{1}{\sqrt{3}-1}$.

3. (i) $Sy = u\mathrm{Sin}\theta t - \frac{1}{2}g\,\mathrm{Cos}\,\alpha t^2 = 0 \Rightarrow t = 0$ or $t = \dfrac{2u\mathrm{Sin}\theta}{g\mathrm{Cos}\alpha}$ = time of flight

(ii) $Sx = u\mathrm{Cos}\theta t - \frac{1}{2}g\mathrm{Sin}\,\alpha t^2$

At $t = \dfrac{2u\mathrm{Sin}\theta}{g\mathrm{Cos}\alpha}$, $Sx = u\mathrm{Cos}\theta\,(\dfrac{2u\mathrm{Sin}\theta}{g\mathrm{Cos}\,\alpha}) - \frac{1}{2}g\,\mathrm{Sin}\,\alpha(\dfrac{4u^2\mathrm{Sin}^2\theta}{g^2\mathrm{Cos}^2\alpha})$

$= \dfrac{2u^2\mathrm{Cos}\theta\mathrm{Sin}\theta\mathrm{Cos}\,\alpha - 2u^2\mathrm{Sin}\alpha\mathrm{Sin}^2\theta}{g\mathrm{Cos}^2\alpha} = \dfrac{2u^2\mathrm{Sin}\theta(\mathrm{Cos}\theta\mathrm{Cos}\alpha - \mathrm{Sin}\,\alpha\mathrm{Sin}\theta)}{g\mathrm{Cos}^2\alpha}$

$= \dfrac{2u^2\mathrm{Sin}\theta\,\mathrm{Cos}(\theta+\alpha)}{g\mathrm{Cos}^2\alpha} = \dfrac{u^2(\mathrm{Sin}(2\theta+\alpha) - \mathrm{Sin}\alpha)}{g\mathrm{Cos}^2\alpha}$

This is a maximum when $\mathrm{Sin}(2\theta + \alpha) = 1 \Rightarrow 2\theta + \alpha = \frac{\pi}{2} \Rightarrow \theta = \frac{1}{2}(\frac{\pi}{2} - \alpha)$ Q.E.D.

3. contd ...

(iii) $Vx = u\mathrm{Cos}\theta - g\mathrm{Sin}\alpha t$.

At $t = \dfrac{2u\mathrm{Sin}\theta}{g\mathrm{Cos}\alpha}$, $Vx = u\mathrm{Cos}\theta - g\,\mathrm{Sin}\,\alpha(\dfrac{2u\mathrm{Sin}\theta}{g\mathrm{Cos}\alpha}) = u\mathrm{Cos}\theta - 2u\mathrm{Tan}\alpha\mathrm{Sin}\theta$.

$Vy = u\mathrm{Sin}\theta - g\mathrm{Cos}\alpha t$.

At $t = \dfrac{2u\mathrm{Sin}\theta}{g\mathrm{Cos}\alpha}$, $Vy = u\mathrm{Sin}\theta - g\mathrm{Cos}\,\alpha(\dfrac{2u\mathrm{Sin}\theta}{g\mathrm{Cos}\alpha}) = -u\mathrm{Sin}\theta$.

Let $\underline{l}$ be the angle of landing.

$\mathrm{Tan}\alpha = \mathrm{Tan}\,\underline{l}$ since it lands horizontally $\therefore \mathrm{Tan}\,\alpha = \dfrac{-Vy}{Vx} \Rightarrow$

$\mathrm{Tan}\,\alpha = \dfrac{u\mathrm{Sin}\theta}{u\mathrm{Cos}\theta - 2u\mathrm{Tan}\,\alpha\mathrm{Sin}\theta} \Rightarrow \mathrm{Tan}\alpha = \dfrac{\mathrm{Sin}\theta}{\mathrm{Cos}\theta - 2\mathrm{Tan}\,\alpha\mathrm{Sin}\theta}$

$\Rightarrow \mathrm{Tan}\alpha\mathrm{Cos}\theta - 2\mathrm{Tan}^2\alpha\,\mathrm{Sin}\theta = \mathrm{Sin}\theta$. Divide by $\mathrm{Cos}\theta \Rightarrow \mathrm{Tan}\alpha - 2\,\mathrm{Tan}^2\alpha\,\mathrm{Tan}\,\theta = \mathrm{Tan}\,\theta$

$\Rightarrow \mathrm{Tan}\,\alpha = \mathrm{Tan}\,\theta\,(1 + 2\mathrm{Tan}^2\alpha) \Rightarrow \mathrm{Tan}\,\theta = \dfrac{\mathrm{Tan}\,\alpha}{1+2\,\mathrm{Tan}^2\alpha} =$

$\dfrac{\mathrm{Sin}\alpha/\mathrm{Cos}\,\alpha}{1+2\mathrm{Sin}^2\alpha\ /\mathrm{Cos}^2\alpha} = \dfrac{\mathrm{Sin}\,\alpha\mathrm{Cos}\,\alpha}{\mathrm{Cos}^2\alpha + 2\mathrm{Sin}^2\alpha} = \dfrac{\mathrm{Sin}\alpha\mathrm{Cos}\,\alpha}{\mathrm{Cos}^2\alpha + 2(1-\mathrm{Cos}^2\alpha)} = \dfrac{\mathrm{Sin}\alpha\,\mathrm{Cos}\alpha}{2-\mathrm{Cos}^2\alpha}$ q.e.d

4. $Sy = 0 \Rightarrow u\mathrm{Sin}\,(\alpha - 45^\circ)t - \frac{1}{2}g\mathrm{Cos}45^\circ t^2 = 0 \Rightarrow u\,\mathrm{Sin}(\alpha - 45^\circ)t - \dfrac{1}{2\sqrt{2}}g\,t^2 = 0 \Rightarrow t = 0$ or $t = \dfrac{2\sqrt{2}u\mathrm{Sin}(\alpha - 45^\circ)}{g}$ = time of flight.

$Vy = u\mathrm{Sin}(\alpha - 45^\circ) - \dfrac{g}{\sqrt{2}}t$. At time of flight, $Vy = u\mathrm{Sin}(\alpha - 45^\circ) - \dfrac{g}{2}(\dfrac{2\ 2u\mathrm{Sin}(\alpha - 45^\circ)}{g}) = -u\mathrm{Sin}(\alpha - 45^\circ)$

$Vx = u\mathrm{Cos}(\alpha - 45^\circ) - \dfrac{g}{\sqrt{2}}t$.

At time of flight, $Vx = u\mathrm{Cos}(\alpha - 45^\circ) - \dfrac{g}{\sqrt{2}}(\dfrac{2\sqrt{2}u\mathrm{Sin}(\alpha - 45^\circ)}{g}) = u\mathrm{Cos}(\alpha - 45^\circ) - 2u\mathrm{Sin}(\alpha - 45^\circ)$. $\mathrm{Tan}\,\beta = \dfrac{-Vy}{Vx} = \dfrac{u\mathrm{Sin}(\alpha - 45^\circ)}{u\mathrm{Cos}(\alpha - 45^\circ) - 2u\mathrm{Sin}(\alpha - 45^\circ)}$

$= \dfrac{\mathrm{Tan}(\alpha - 45^\circ)}{1-2\mathrm{Tan}(\alpha - 45^\circ)}$. But $\mathrm{Tan}\,(\alpha - 45^\circ) = \dfrac{\mathrm{Tan}\,\alpha - \mathrm{Tan}45^\circ}{1 + \mathrm{Tan}\,\alpha\mathrm{Tan}\,45^\circ} = \dfrac{\mathrm{Tan}\alpha - 1}{1 + \mathrm{Tan}\alpha}$

$\therefore \mathrm{Tan}\,\beta = \dfrac{\frac{\mathrm{Tan}\alpha - 1}{1+\mathrm{Tan}\alpha}}{1 - 2\left(\frac{\mathrm{Tan}\alpha - 1}{1+\mathrm{Tan}\alpha}\right)} = \dfrac{\mathrm{Tan}\alpha - 1}{1 + \mathrm{Tan}\,\alpha - 2\mathrm{Tan}\alpha + 2} = \dfrac{\mathrm{Tan}\alpha - 1}{3-\mathrm{Tan}\alpha}$

(i) If it lands horizontally, then $\beta = 45^\circ \Rightarrow 1 = \dfrac{\mathrm{Tan}\alpha - 1}{3 - \mathrm{Tan}\alpha}$

$\Rightarrow 3 - \mathrm{Tan}\,\alpha = \mathrm{Tan}\alpha - 1 \Rightarrow \mathrm{Tan}\,\alpha = 2$.

(ii) If it lands vertically, then $\beta = 90^\circ \Rightarrow \dfrac{\mathrm{Tan}\alpha - 1}{3 - \mathrm{Tan}\,\alpha}$ = undefined

$Sy = 0 \Rightarrow u \sin\beta\, t - \frac{1}{2}g \cos\alpha\, t^2 = 0 \Rightarrow \frac{1}{\sqrt{5}}\, ut - \frac{2}{5} g t^2 = 0$

$\Rightarrow t = 0$ or $t = \frac{\sqrt{5}u}{2g}$ = time of flight

$Vy = u \sin\beta - g\cos\alpha\, t = \frac{1}{\sqrt{5}} u - \frac{4}{5} gt$

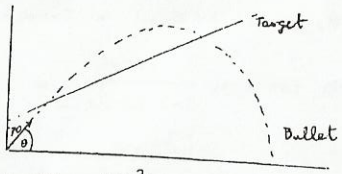
At $t = \frac{\sqrt{5}u}{2g}$, $Vy = \frac{1}{\sqrt{5}}u - \frac{4}{5}g\left(\frac{\sqrt{5}u}{2g}\right) = -\frac{u}{\sqrt{5}}$

$Vx = u\cos\beta - g \sin\alpha\, t = \frac{2}{\sqrt{5}}u - \frac{3}{5}gt$.

At $t = \frac{\sqrt{5}u}{2g}$, $Vx = \frac{2}{\sqrt{5}}u - \frac{3}{5}g\left(\frac{\sqrt{5}u}{2g}\right) = \frac{2u}{\sqrt{5}} - \frac{3u}{2\sqrt{5}} = \frac{u}{2\sqrt{5}}$

$\tan \underline{1} = \frac{-Vy}{Vx} = \frac{u/\sqrt{5}}{u/2\sqrt{5}} = 2 \therefore \underline{1} = \tan^{-1} 2$

7.



Bullet: $Sx = 70\cos\theta t = 70(\frac{3}{5})t = 42t$

$Sy = 70\sin t - \frac{1}{2}gt^2 = 70(\frac{4}{5})t - \frac{1}{2}(9.8)t^2 = 56t - 4.9t^2$

Target: $Sx = 42\sqrt{2} \cos 45^\circ t = 42t$

$Sy = 10 + 42\sqrt{2} \sin 45^\circ t = 10 + 42t$

At moment of striking $56t - 4.9t^2 = 10 + 42t$

$\Rightarrow 49t^2 - 140t + 100 = 0$

$\Rightarrow (7t - 10)(7t - 10) = 0$

$\Rightarrow t = 10/7$

$Sx = 42t = 42(\frac{10}{7}) = 60$m from 0

Bullet: $Sy = 70 \sin\theta\, t - 49.t^2$

Target: $Sy = 10 + 42t$

Intersection: $70 \sin\theta\, t - 4.9t^2 = 10 + 42t \Rightarrow 49t^2 + (420 - 70\sin\theta)t + 100 = 0$

Real Values $\Rightarrow b^2 \geq 4ac \Rightarrow (420 - 700\sin\theta)^2 \geq 19600$

$\Rightarrow \sin\theta \geq 4/5$ (or $\sin\theta \leq -4/5$: reject) $\Rightarrow \tan\theta \geq 4/3$

(In fact for Sx = Sx, Cos θ must be equal to 3/5 and so Tan θ must be 4/3)

4. (ii) contd.

$\Rightarrow 3 - \tan\alpha = 0 \Rightarrow \tan\alpha = 3$.

(iii) If $\tan\beta = \frac{1}{3}$, $\frac{1}{3} = \frac{\tan\alpha - 1}{3 - \tan\alpha} \Rightarrow 3 - \tan\alpha = 3\tan\alpha - 3$

$\Rightarrow \tan\alpha = \frac{3}{2}$

5. $Sy = 0 \Rightarrow u\sin\alpha t - \frac{1}{2}gt^2 = 0 \Rightarrow t = 0$ or $t = \frac{2u\sin\alpha}{g}$ = time of flight

At the time of flight, $Sx = u\cos\alpha t = u\cos\alpha\left(\frac{2u\sin\alpha}{g}\right) = \frac{2u^2\cos\alpha\sin\alpha}{g}$

When Sx = 3, Sy = 1 $\Rightarrow u\cos\alpha t = 3$ when $u \sin\alpha\, t - \frac{1}{2}gt^2 = 1$

$\therefore t = \frac{3}{u\cos\alpha}$ can be put into this second equation.

$u\sin\alpha\left(\frac{3}{u\cos\alpha}\right) - \frac{1}{2}g\left(\frac{9}{u^2\cos^2\alpha}\right) = 1 \Rightarrow 3\tan\alpha - \frac{9g}{2u^2\cos^2\alpha} = 1$

... Equation A

When Sx = 1, Sy = 3 $\Rightarrow u\cos\alpha t = 1$ when $u \sin\alpha t - \frac{1}{2}gt^2 = 3$

$t = \frac{1}{u\cos\alpha}$ can be put into this second equation:

$u\sin\alpha\left(\frac{1}{u\cos\alpha}\right) - \frac{1}{2}g\left(\frac{1}{u^2\cos^2\alpha}\right) = 3 \Rightarrow \tan\alpha - \frac{g}{2u^2\cos^2\alpha} = 3$ ... Equation B

9B : $9\tan\alpha - \frac{9g}{2u^2\cos^2\alpha} = 27$ and -A: $-3\tan\alpha + \frac{9g}{2u^2\cos^2\alpha} = -1$

Adding these gives the result $6\tan\alpha = 26$

$\Rightarrow \tan\alpha = \frac{13}{3} \Rightarrow \sin\alpha = \frac{13}{\sqrt{178}}$, $\cos\alpha = \frac{3}{\sqrt{178}}$

Putting these results into B gives:

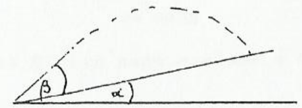
$\frac{13}{3} - \frac{g}{2u^2(9/178)} = 3 \Rightarrow u^2 = \frac{89g}{12}$

Now the range was found to be $\frac{2u^2\cos\alpha\sin\alpha}{g}$

$\therefore$ Range $= 2\left(\frac{89g}{12}\right)\left(\frac{3}{\sqrt{178}}\right)\left(\frac{13}{\sqrt{178}}\right)\left(\frac{1}{g}\right) = \frac{13}{4}$.

6. $\sin\alpha = \frac{3}{5}$, $\cos\alpha = \frac{4}{5}$

$\sin\beta = \frac{1}{\sqrt{5}}$, $\cos\beta = \frac{2}{\sqrt{5}}$



8. $Sx = u\cos\theta t = \frac{4}{5}ut$

$Sy = u\sin\theta t - \frac{1}{2}gt^2 = \frac{3}{5}ut - 4.9t^2$. When $Sx = 240$, $Sy \geqslant 90$

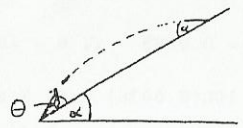
$Sx = 240 \Rightarrow \frac{4}{5}ut = 240 \Rightarrow t = \frac{300}{u}$

$Sy \geqslant 90 \Rightarrow \frac{3}{5}u\left(\frac{300}{u}\right) - 4.9\left(\frac{9000}{u^2}\right) \geqslant 90 \Rightarrow 180 - \frac{441000}{u^2} \geqslant 90$

$\Rightarrow u^2 \geqslant 4900 \Rightarrow u \geqslant 70$.

Answer: 70 m/s .

9. $\sin\alpha = \frac{1}{\sqrt{5}}$



$\cos\alpha = \frac{2}{\sqrt{5}}$

$Sy = 0 \Rightarrow u\sin\theta t - \frac{1}{2}g\cos\alpha t^2 = 0 \Rightarrow u\sin\theta t - \frac{1}{\sqrt{5}}g t^2 = 0$

$\Rightarrow t = 0$ or $t = \frac{\sqrt{5}u\sin\theta}{g}$ = time of flight .

$Vy = u\sin\theta - g\cos\alpha t = u\sin\theta - \frac{2}{\sqrt{5}}gt$

At time of flight, $Vy = u\sin\theta - \frac{2}{\sqrt{5}}g\left(\frac{\sqrt{5}u\sin\theta}{g}\right) = -u\sin\theta$

$Vx = u\cos\theta - g\sin\alpha t = u\cos\theta - \frac{1}{\sqrt{5}}gt$

At time of flight, $Vx = u\cos\theta - \frac{1}{\sqrt{5}}g\left(\frac{\sqrt{5}u\sin\theta}{g}\right) = u\cos\theta - u\sin\theta$

Since the particle lands horizontally,

$\tan\underline{1} = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{-Vy}{Vx} \Rightarrow \frac{1}{2} = \frac{u\sin\theta}{u\cos\theta - u\sin\theta} \Rightarrow 2u\sin\theta = u\cos\theta - u\sin\theta$

$\Rightarrow \tan\theta = \frac{1}{3}$

$\tan\theta = \frac{1}{3} \Rightarrow \sin\theta = \frac{1}{\sqrt{10}}$, $\cos\theta = \frac{3}{\sqrt{10}}$. Now $Sx = u\cos\theta t - \frac{1}{2}g\sin\alpha t^2$

$= \frac{3u}{\sqrt{10}}t - \frac{1}{2\sqrt{5}}gt^2$. At time of flight, $t = \frac{\sqrt{5}u\sin\theta}{g} = \frac{\sqrt{5}u}{\sqrt{10}g} = \frac{u}{\sqrt{2}g}$

At $t = \frac{u}{\sqrt{2}g}$, $Sx = \frac{3u}{\sqrt{10}}\left(\frac{u}{\sqrt{2}g}\right) - \frac{1}{2\sqrt{5}}g\left(\frac{u^2}{2g^2}\right) = \frac{3u^2}{2\sqrt{5}g} - \frac{u^2}{4\sqrt{5}g} = \frac{6u^2 - u^2}{4\sqrt{5}g}$

$= \frac{5u^2}{4\sqrt{5}g} = \frac{\sqrt{5}u^2}{4g}$

CHAPTER 4 RELATIVE VELOCITY

Exercise 4.A

1. $\vec{V}_{BA} = 30\vec{i} - 25\vec{i} = 5\vec{i}$ m/s . $\frac{1000}{5} = 200$ seconds.

2. $\vec{V}_c = 10\vec{i}$, $\vec{V}_L = -15\vec{i}$ $\therefore \vec{V}_{cL} = 10\vec{i} - (-15\vec{i}) = 25\vec{i}$ m/s.

$\frac{500}{25} = 20$ seconds.

3. $\vec{V}_{pq} = \vec{V}_p - \vec{V}_q = (5\vec{i} + 2\vec{j}) - (2\vec{i} - 2\vec{j}) = 3\vec{i} + 4\vec{j}$

$|\vec{V}_{pq}| = \sqrt{3^2 + 4^2} = 5$km/hr. $\frac{20}{5} = 4$ hours.

4. $\vec{V}_{AB} = (4\vec{i} - 3\vec{j}) - (6\vec{i} - \vec{j}) = -2\vec{i} - 2\vec{j}$ m/s.

$|\vec{V}_{AB}| = \sqrt{4 + 4} = \sqrt{8}$ m/s .

Direction = SW . $V_{cB} = 8\vec{i} - (6\vec{i} - \vec{j}) = 2\vec{i} + \vec{j}$ m/s.

$|\vec{V}_{cB}| = \sqrt{4 + 1} = \sqrt{5}$ m/s .

$\tan\theta = \frac{1}{2} \Rightarrow \theta = 26^\circ 34'$.

Direction : E 26° 34'N .

5. $\vec{r}_{BA} = (-3\vec{i} + 6\vec{j}) - (4\vec{i} + 2\vec{j}) = -7\vec{i} + 4j$.

$\vec{r}_{CA} = (-4\vec{i} + 2\vec{j}) - (4\vec{i} + 2\vec{j}) = -8\vec{i}$.

$|\vec{r}_{BA}| = \sqrt{49 + 16} = \sqrt{65}$.

$|\vec{r}_{CA}| = \sqrt{64} = 8$.

Since $|\vec{r}_{BA}| > |\vec{r}_{CA}|$, B is farther .

6. $\vec{r}_{QP} = (-4\vec{i} + \vec{j}) - (\vec{i} - 2\vec{j}) = -5\vec{i} + 3\vec{j}$.

Let $\vec{r}_T = a\vec{i} + b\vec{j}$. $\vec{r}_{TS} = \vec{r}_{QP} = (a + 3)\vec{i} + (b - 5)\vec{j} = -5\vec{i} + 3\vec{j}$.

$\Rightarrow a + 3 = -5$ and $b - 5 = 3 \Rightarrow a = -8$ and $b = 8$ $\therefore \vec{r}_T = -8\vec{i} + 8\vec{j}$.

7. $\vec{V}_{CT} = \vec{V}_c - \vec{V}_T = 10\vec{i} + 6\vec{j} - 30\vec{j} = 10\vec{i} - 24\vec{j}$.

$|\vec{V}_{CT}| = \sqrt{100 + 576} = 26$ m/s . $\tan\theta = \frac{24}{10} = 2.4 \Rightarrow \theta = 67^\circ 23'$

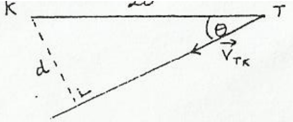
Direction : E 67° 23'S .

8. $\vec{V}_{QP} = (-4\vec{i} + 2\vec{j}) - (6\vec{i} + 2\vec{j}) = 10\vec{i}$ m/s.

Time = $\frac{100}{10} = 10$ seconds .

2

9.

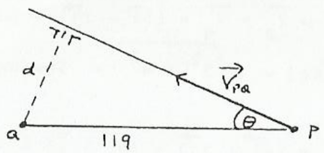


$V_{TK} = (-2\vec{i} - 2\vec{j}) - (\vec{i} + 2\vec{j}) = -3\vec{i} - 4\vec{j}$.

$\tan\theta = \frac{-4}{-3} = \frac{4}{3} \Rightarrow \sin\theta = \frac{4}{5}$.

$d = 20 \sin\theta = 20\ (\frac{4}{5}) = 16m$.

10. $V_{PQ} = (-8\vec{i} + 12\vec{j}) - (7\vec{i} + 4\vec{j}) = -15\vec{i} + 8\vec{j}$.

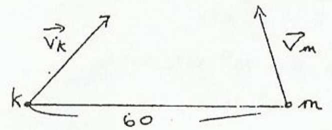


$\tan\theta = \frac{8}{15} \Rightarrow \sin\theta = \frac{8}{17}$.

$d = 119 \sin\theta = 119(\frac{8}{17}) = 56$ units.

11. (a) $\sqrt{t^2 + 9} = 5 \Rightarrow t = 4$.

(b) (i)



$\vec{V}_m = -2\vec{i} + 3\vec{j}$

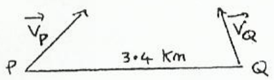
$\vec{V}_k = t\vec{i} + 3\vec{j}$

But $|t\vec{i} + 3\vec{j}| = 5 \Rightarrow t = 4$, as before $\therefore \vec{V}_k = 4\vec{i} + 3\vec{j}$.

(ii) $\vec{V}_{mK} = (-2\vec{i} + 3\vec{j}) - (4\vec{i} + 3\vec{j}) = -6\vec{i}$.

$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{60}{6} = 10$ hours.

12.



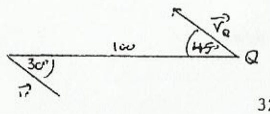
$\vec{V}_P = 5\vec{i} + 5\vec{j}$, $\vec{V}_Q = K\vec{i} + 5\vec{j}$.

But $|K\vec{i} + 5\vec{j}| = 13 \Rightarrow K = -12$ (it must be negative so that Q approaches P)

$\therefore \vec{V}_Q = -12\vec{i} + 5\vec{j}$. $V_{QP} = (-12\vec{i} + 5\vec{j}) - (5\vec{i} + 5\vec{j}) = -17\vec{j}$,

$\therefore |\vec{V}_{QP}| = 17$ km/hr. $\text{Time} = \frac{3400}{17} = 200s$.

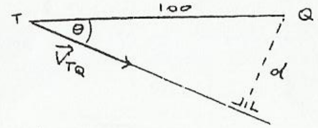
13.



13 contd..

$\vec{V}_T = 8.66\vec{i} - 5\vec{j}$. $\vec{V}_Q = -14.14\vec{i} + 14.14\vec{j}$. $\vec{V}_{TQ} = \vec{V}_T - \vec{V}_Q = 22.80\vec{i} - 19.14\vec{j}$.

$|\vec{V}_{TQ}| = \sqrt{(22.8)^2 + (-19.14)^2} = 29.77$



$\tan\theta = \frac{19.14}{22.8} = 0.8395 \quad \therefore \theta = 40^\circ\ 1'$.

$d = 100 \sin\theta = 100(0.6430) = 64.3$ km.

EXERCISE 4.8

1. $\vec{V}_B = \vec{V}_{BR} + \vec{V}_R = 8\vec{j} + 6\vec{i} = 6\vec{i} + 8\vec{j}$ m/s.

$V_B = \sqrt{6^2 + 8^2} = 10$ m/s. $\tan\theta = \frac{8}{6} = 1.3333 \Rightarrow \theta = 53^\circ 8'$.

Direction : E 53° 8' N .

After 3 seconds it will be 8 x 3 = 24 m from the bank and 6 x 3 = 18 m downstream.

2. $\vec{V}_B = \vec{V}_{BR} + \vec{V}_R = 12\vec{j} + 5\vec{i} = 5\vec{i} + 12\vec{j}$.

$|\vec{V}_B| = \sqrt{25 + 144} = \sqrt{169} = 13$ m/s .

$\tan\theta = \frac{12}{5} = 2.4 \Rightarrow \theta = 67^\circ\ 23'$.

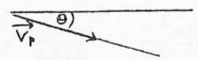
Direction is E67° 23' N. $\text{Time} = \frac{60}{12} = 5$ seconds.

Distance downstream = 5 x 5 = 25m.

3. $\vec{V}_P = \vec{V}_{PW} + \vec{V}_W = 100\vec{i} - 10\vec{j}$ km/hr.

$\vec{V}_P = \sqrt{100^2 + 10^2} = 100.5$ km/hr.

$\tan\theta = \frac{10}{100} = 0.1 \Rightarrow \theta = 5^\circ\ 43'$.



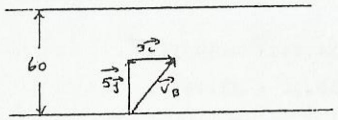
Direction : E 5° 43' S.

4. Upstream: $\vec{V}_C = \vec{V}_{CR} + \vec{V}_R = -5\vec{i} + (3\vec{i}) = -2\vec{i}$. $\text{TIME} = \frac{80}{2} = 40$ seconds.

Downstream : $\vec{V}_C = 5\vec{i} + 3\vec{i} = 8\vec{i}$.

$\text{TIME} = \frac{80}{8} = 10$ seconds.

8. (i) $\vec{V}_{BR} = 5\vec{j}$.

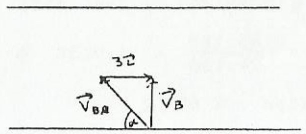


$\vec{V}_B = 3\vec{i} + 5\vec{j}$

Time taken $= \frac{60}{5} = 12$ seconds

(ii) $\vec{V}_{BR} = -5\cos\alpha\ \vec{i} + 5\sin\alpha\vec{j}$

$\vec{V}_B = (-5\cos\alpha + 3)\vec{i} + 5\sin\alpha\vec{j}$



$\therefore -5\cos\alpha + 3 = 0$

$\Rightarrow \cos\alpha = 3/5 \Rightarrow \sin\alpha = 4/5$.

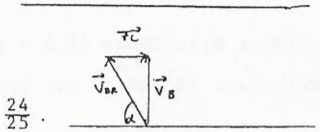
$V_B = 0\vec{i} + 5(\frac{4}{5})\vec{j} = 0\vec{i} + 4\vec{j}$.

Time $= \frac{60}{4} = 15$ seconds .

9.

(i) $V_R = \vec{7i}$.

$\vec{V}_{BR} = -25\cos\alpha\vec{i} + 25\sin\alpha\vec{j}$.



$\therefore \vec{V}_B = (7 - 25\cos\alpha)\vec{i} + 25\sin\alpha\vec{j}$.

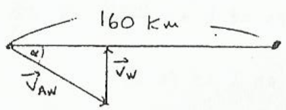
$7 - 25\cos\alpha = 0 \Rightarrow \cos\alpha = \frac{7}{25} \Rightarrow \sin\alpha = \frac{24}{25}$.

Since $\cos\alpha = \frac{7}{25} = 0.28$, $\alpha = 73^\circ 44'$.

(ii) $V_B = 0\vec{i} + 25(\frac{24}{25})\vec{j} = 24\vec{j}$.

Time $= \frac{120}{24} = 5$ seconds .

10. $V_w = 60\vec{j}$.



$V_{AW} = 100\cos\alpha\vec{i} - 100\sin\alpha\ \vec{j}$.

$\therefore V_A = 100\cos\alpha\vec{i} + (60 - 100\sin\alpha)\vec{j}$,

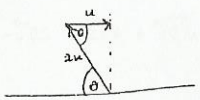
But $60 - 100\sin\alpha = 0 \Rightarrow \sin\alpha = \frac{3}{5} \Rightarrow \cos\alpha = \frac{4}{5}$.

$\therefore V_A = 100\ (\frac{4}{5})\vec{i} + 0\vec{j} = 80\vec{i}$.

Time $= \frac{160}{80} = 2$ hours .

Yes, because of symmetry.

11.



$\cos\theta = \frac{u}{2u} = \frac{1}{2} \Rightarrow \theta = 60^\circ$.

12. $\vec{V}_c = -\vec{j}$.

$\vec{V}_s = 2\cos 45^\circ\vec{i} - 2\sin 45^\circ\vec{j} = 1.414\vec{i} - 1.414\vec{j}$.

$\vec{V}_{sc} = \vec{V}_s - \vec{V}_c = 1.414\vec{i} - 0.414\vec{j}$.

$\vec{V}_{sc} = \sqrt{(1.414)^2 + (-0.414)^2} = 1.47$ m/s .

13. $\vec{V}_A = -100\vec{i}$.

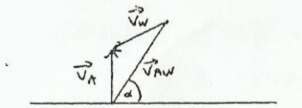
$\vec{V}_W = -20\cos 30\vec{i} + 20\sin 30\vec{j} = -17.32\vec{i} + 10\vec{j}$.

$\vec{V}_{AW} = \vec{V}_A - \vec{V}_W = -100\vec{i} - (17.32\vec{i} + 10\vec{j}) = -82.68\vec{i} - 10\vec{j}$.

$|\vec{V}_{AW}| = \sqrt{(-82.68)^2 + (-10)^2} = 83.28$ km/hr .

14. $\vec{V}_w = -50\cos 45^\circ\vec{i} - 50\sin 45^\circ\vec{j} = -35.355\ \vec{i} - 35.355\vec{j}$.

$\vec{V}_{Aw} = 200\cos\alpha\vec{i} + 200\sin\alpha\ \vec{j}$.



$\vec{V}_A = (200\cos\alpha - 35.355)\vec{i} + (200\sin\alpha - 35.355)\vec{j}$.

But $200\cos\alpha - 35.355 = 0 \Rightarrow \cos\alpha = \frac{35.355}{200} = 0.1768 \Rightarrow \alpha = 79^\circ 49'$

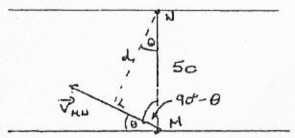
$\therefore \vec{V}_A = 0\vec{i} + (200(0.9843) - 35.355)\vec{j} = 161.505j$.

Answer: 161.5 km/hr .

15. $\vec{V}_M = 4\vec{i}$, $\vec{V}_N = \frac{5}{3}j$, $\vec{V}_{MN} = \vec{V}_M - \vec{V}_N = 4\vec{i} - \frac{5}{3}\vec{j}$.

$|\vec{V}_{MN}| = \sqrt{16 + \frac{25}{9}} = \frac{13}{3}$ m/s .

$\tan\theta = \frac{5/3}{4} = \frac{5}{12} = 0.4167 \Rightarrow \theta = 22^\circ 37'$. Direction is E 22° 37' S .



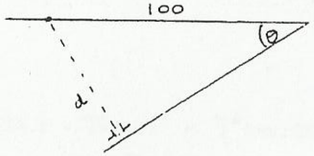
$d = 50\cos\theta = 50(0.9231) = 46.155$m (or $50(\frac{12}{13}) = \frac{600}{13}$ m)

16. $\vec{V}_{AB} = a\vec{i} - b\vec{j} \Rightarrow |\vec{V}_{AB}| = \sqrt{a^2 + b^2}$

$\vec{V}_x = 7\vec{i}$, $\vec{V}_y = -24\vec{j}$ $\therefore$ $\vec{V}_{yx} = -24\vec{j} - 7\vec{i} = -7\vec{i} - 24\vec{j}$

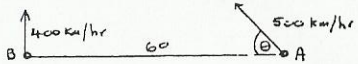
16. contd ..

Tan $\theta = \frac{24}{7} \Rightarrow$ Sin $\theta = \frac{24}{25}$.



$\therefore d = 100$ Sin $\theta = 100(\frac{24}{25}) = 96$m .

17. (i)



(ii) $\vec{V}_{AB}$ = Due West .

(iii) $\vec{V}_A = K\vec{i} + 400\vec{j}$.

$|K\vec{i} + 400\vec{j}| = 500 \Rightarrow K = -300$ (so that A approaches B).

$\therefore \vec{V}_A = -300\vec{i} + 400\vec{j}$

Tan $\theta = \frac{400}{300} = 1.3333 \Rightarrow \theta = 53^\circ\ 8'$,

Direction is W $53^\circ\ 8'$ N .

(iv) $\vec{V}_{AB} = (-300\vec{i} + 400\vec{j}) - 400\vec{j} = -300\vec{i} \therefore |\vec{V}_{AB}| = 300$ km/h .

(v) Time = $\frac{60}{300} = \frac{1}{5}$ hr = 12 minutes .

Time is 15.12 hours.

18. $\vec{V}_D = 14.142\,\vec{i} + 14.142\vec{j}$. $\vec{V}_{CD} = 10\vec{i}$.

$\therefore \vec{V}_C = \vec{V}_{CD} + \vec{V}_D = 24.142\vec{i} + 14.142\vec{j}$.

$|\vec{V}_C| = \sqrt{(24.142)^2 + (14.142)^2} = 28$ km/hr.

Tan $\theta = \frac{14.142}{24.142} = 0.5858 \Rightarrow \theta = 30^\circ$.

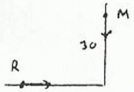
Direction: E 30° N .

19. $\vec{V}_{PQ} = (3\vec{i} - \vec{j}) - (-2\vec{i} + 7\vec{j}) = 5\vec{i} - 8\vec{j}$.

$\vec{r}_{PQ} = -10\vec{i} - (-16\vec{j}) = -10\vec{i} + 16\vec{j} = -2\vec{V}_{PQ}$.

$\therefore$ They are on collision course.

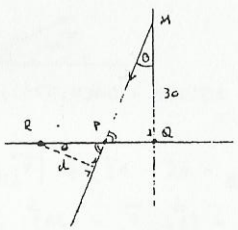
20.



$\vec{V}_m = -24\vec{j}$. $\vec{V}_r = 10\vec{i}$.

$\vec{V}_{mr} = -24\vec{j} - 10\vec{i} = -10\vec{i} - 24\vec{j}$

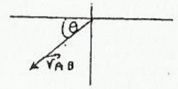
Tan $\theta = \frac{10}{24} = \frac{5}{12} \Rightarrow$ Cos $\theta = \frac{12}{13}$, Sin $\theta = \frac{5}{13}$.



Tan $\theta = \frac{|PQ|}{30} \Rightarrow \frac{5}{12} = \frac{|PQ|}{30} \Rightarrow |PQ| = 12\frac{1}{2}$.

$|RP| = 20 - 12\frac{1}{2} = 7\frac{1}{2}$. Now $d = |RP|$ Cos $\theta = (7\frac{1}{2})(\frac{12}{13}) = \frac{90}{13}$

21. $\vec{V}_A = 14.142\vec{i} - 14.142\vec{j}$.



$\vec{V}_B = 38.3\vec{i} + 32.14\vec{j}$.

$\vec{V}_{AB} = (14.14\vec{i} - 14.142\vec{j}) - (38.3\vec{i} + 32.14\,\vec{j}) = -24.158\vec{i} - 46.282\vec{j}$.

$|\vec{V}_{AB}| = \sqrt{(-24.158)^2 + (-46.282)^2} = 52$ km/hr .

Tan $\theta = \frac{46.282}{24.158} = 1.9158 \Rightarrow \theta = 62^\circ$.

Direction : W 62° S .

Let X = new speed of B.

$\vec{V}_B = 0.766X\vec{i} + 0.6428X\vec{j}$. $\vec{V}_A = 14.142\vec{i} - 14.142\vec{j}$.

$\vec{V}_{AB} = (14.142 - 0.766X)\vec{i} + (-14.142 - 0.6428X)\vec{j}$

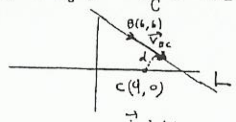
The $\vec{i}$ - component must be zero. $\therefore x = \frac{14.142}{0.766} = 18.5$

It must reduce from 50 to 18.5 - that is a reduction of 31.5 km/hr

Note: Questions 22 and 23 may be solved using trigonometry.

22. The positions are: Bus B : (6,0); Car C: (9,0)

$\vec{V}_B = -4\vec{i} - 4\vec{j}$. $\vec{V}_C = -7\vec{i}$. $\vec{V}_{BC} = -4\vec{i} - 4\vec{j} + 7\vec{i} = 3\vec{i} - 4\vec{j}$



$\left.\begin{array}{l}\text{The slope of L} = \frac{\vec{j}\text{-bit}}{\vec{i}\text{-bit}} = \frac{-4}{3} = -\frac{4}{3} \\ \text{A point on L is } (6,6)\end{array}\right\}$ Equation is $y - 6 = -\frac{4}{3}(x - 6)$ $\Rightarrow 4x + 3y - 42 = 0$

The distance from (9,0) to $4x + 3y - 42 = 0$ is

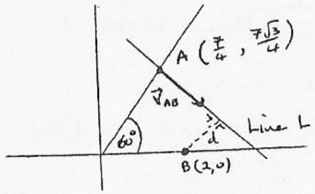
$$\frac{|36 + 0 - 42|}{\sqrt{4^2 + 3^2}} = \frac{6}{5} = 1.2 \text{ km}$$

23. Position of A: $(\frac{7}{2}$ Cos 60°, $\frac{7}{2}$ Sin $60^\circ) = (\frac{7}{4}, \frac{7\sqrt{3}}{4})$.

Position of B: (2,0) .

$\vec{V}_A = -15\vec{i} - 15\sqrt{3}\vec{j}$. $V_B = -40\vec{i}$. $V_{AB} = -15\vec{i} - 15\sqrt{3}\vec{j} + 40\vec{i} = 25\vec{i} - 15\sqrt{3}\vec{j}$

23. contd ...

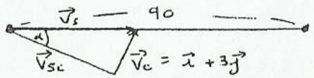


Slope of L = $\dfrac{-15\sqrt{3}}{25} = \dfrac{-3\sqrt{3}}{5}$
A point on L is $(\frac{7}{4}, \frac{7\sqrt{3}}{4})$ } Equation of L is $y - \frac{7\sqrt{3}}{4} = \frac{-3\sqrt{3}}{5}(x - \frac{7}{4})$
$\Rightarrow 3\sqrt{3}x + 5y - 14\sqrt{3} = 0$.

The distance from (2,0) to $3\sqrt{3X} + 5y - 14\sqrt{3} = 0$ is given by:

$$\frac{|6\sqrt{3} + 0 - 14\sqrt{3}|}{\sqrt{(3\sqrt{3})^2 + 5^2}} = \frac{8\sqrt{3}}{\sqrt{52}} = 1.92 .$$

24.

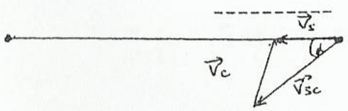


$\vec{V}_{SC} = 5\cos\alpha\vec{i} - 5\sin\alpha\vec{j}$. $V_C = \vec{i} + 3\vec{j}$.
$\vec{V}_S = \vec{V}_{SC} + \vec{V}_C = (5\cos\alpha + 1)\vec{i} + (-5\sin\alpha + 3)\vec{j}$.

$\vec{j}$ - component = 0 $\Rightarrow \sin\alpha = \frac{3}{5} \Rightarrow \cos\alpha = \frac{4}{5}$.

$\therefore \vec{V}_S = (5(\frac{4}{5}) + 1)\vec{i} + 0\vec{j} = 5\vec{i}$.

Time = $\frac{90}{5}$ = 18 seconds .

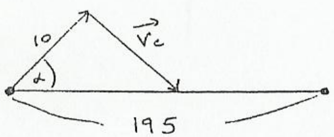


$\vec{V}_{SC} = -5\cos\alpha\,\vec{i} - 5\sin\alpha\vec{j}$. $\vec{V}_C = \vec{i} + 3\vec{j}$. $V_S = (-5\cos\alpha + 1)\vec{i} + (-5\sin\alpha + 3)\vec{j}$

Again, $-5\sin\alpha + 3 = 0 \Rightarrow \sin\alpha = \frac{3}{5} = \cos\alpha \Rightarrow \frac{4}{5}$ $\therefore V_S = (-5(\frac{4}{5}) + 1)\vec{i} + 0\vec{j}$
$= -3\vec{i}$

Time = $\frac{90}{3}$ = 30 seconds .

25.



25. contd ...

$\vec{V}_{SC} = 10\cos\alpha\,\vec{i} + 10\sin\alpha\vec{j}$. $V_C = 5\vec{i} - 6\vec{j}$
$\vec{V}_S = \vec{V}_{SC} + \vec{V}_C = (10\cos\alpha + 5)\vec{i} + (10\sin\alpha - 6)\vec{j}$

$\vec{j}$ - component = 0 $\Rightarrow 10\sin\alpha - 6 = 0 \Rightarrow \sin\alpha = \frac{3}{5} \Rightarrow \cos\alpha = \frac{4}{5}$

$\therefore \vec{V}_S = (10(\frac{4}{5}) + 5)\vec{i} + 0\vec{j} = 13\vec{i}$. TIME = $\frac{195}{13}$ = 15 seconds

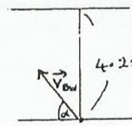
Returning is similar, giving the result
$\vec{V}_S = (-10\cos\alpha + 5)\vec{i} + (10\sin\alpha - 6)\vec{j}$
$10\sin\alpha - 6 = 0 \Rightarrow \sin\alpha = \frac{3}{5} \Rightarrow \cos\alpha = \frac{4}{5}$

$\therefore \vec{V}_S = (-10(\frac{4}{5}) + 5)\vec{i} = -3\vec{i}$. TIME = $\frac{195}{3}$ = 65 seconds

$\therefore$ Total time = 15 + 65 = 80 seconds

The difference between the outward and return speeds must be 2 x 5 = 10 m/s
(Since outward speed gains 5m/s from the current, but return speed loses 5m/s)
The outward speed = $\frac{195}{13}$ = 15 m/s, the return speed will be 15 - 10 = 5 m/s
The time will be $\frac{195}{5}$ = 39 seconds.

26.



$\vec{V}_{BW} = -18\cos\alpha\vec{i} + 18\sin\alpha\vec{j}$
$\vec{V}_W = 8\sqrt{2}\vec{i} - 8\sqrt{2}\vec{j}$
$\therefore \vec{V}_B = (-18\cos\alpha + 8\sqrt{2})\vec{i} + (18\sin\alpha - 8\sqrt{2})\vec{j}$

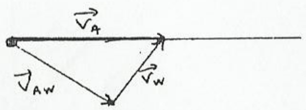
The $\vec{i}$-component is zero $\Rightarrow -18\cos\alpha + 8\sqrt{2} = 0$
$\Rightarrow \cos\alpha = \frac{8\sqrt{2}}{18} = \frac{4\sqrt{2}}{9} \Rightarrow \sin\alpha = \frac{7}{9}$.

$\therefore V_B = 0\vec{i} + (18(\frac{7}{9}) - 8\sqrt{2})\vec{j} = (14 - 8\sqrt{2})\vec{j}$. TIME = $\frac{4.25}{14-8\sqrt{2}}$

Similarily, returning time = $\frac{4.25}{14+8\sqrt{2}}$.

Total time = $\frac{4.25}{14-8\sqrt{2}} + \frac{4.25}{14+8\sqrt{2}} = \frac{4.25(14+8\sqrt{2}) + 4.25(14-8\sqrt{2})}{(14-8\sqrt{2})(14+8\sqrt{2})} = \frac{7}{4}$ hours.

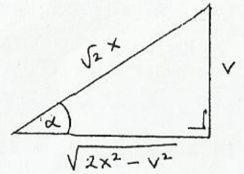
27. (i)



$\vec{V}_W = \frac{V}{\sqrt{2}}\vec{i} + \frac{V}{\sqrt{2}}\vec{j}$. $\vec{V}_{AW} = X\cos\alpha\vec{i} - X\sin\alpha\vec{j}$ $\therefore V_A = (\frac{V}{\sqrt{2}} + X\cos\alpha)\vec{i} + (\frac{V}{\sqrt{2}} - X\sin\alpha)\vec{j}$. $\vec{j}$ - component = 0 $\Rightarrow \frac{V}{2} - X\sin\alpha = 0 \Rightarrow \sin\alpha = \frac{V}{\sqrt{2}X}$

27. contd..

$\therefore \cos\alpha = \dfrac{\sqrt{2X^2 - V^2}}{\sqrt{2}\,X}$



$\therefore \vec{V}_A = \left(\dfrac{V}{2} + \dfrac{X\sqrt{2X^2 - V^2}}{\sqrt{2}\,X}\right)\vec{i} = \left(\dfrac{V}{\sqrt{2}} + \dfrac{\sqrt{2X^2 - V^2}}{\sqrt{2}}\right)\vec{i}$

$\therefore |\vec{V}_A| = \dfrac{V + \sqrt{2X^2 - V^2}}{\sqrt{2}} = U_1$

Similarily $U_2 = \dfrac{\sqrt{2X^2 - V^2} - V}{\sqrt{2}}$

$\therefore U_1 - U_2 = \dfrac{2V}{\sqrt{2}} = \sqrt{2}\,V$ Q.E.D.

(ii) $U_1U_2 = \dfrac{(\sqrt{2X^2 - V^2} + V)}{\sqrt{2}}\dfrac{(\sqrt{2X^2 - V^2} - V)}{\sqrt{2}} = \dfrac{2X^2 - 2V^2}{2} =$

$X^2 - V^2$ Q.E.D.

(iii) $\text{TIME} = \dfrac{\text{DISTANCE}}{\text{SPEED}} = \dfrac{d}{(\sqrt{2X^2 - V^2} + V)/\sqrt{2}} + \dfrac{d}{(\sqrt{2X^2 - V^2} - V)/\sqrt{2}}$

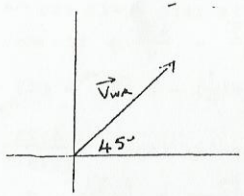
$= \dfrac{\sqrt{2}\,d}{\sqrt{2X^2 - V^2} + V} + \dfrac{\sqrt{2}\,d}{\sqrt{2X^2 - V^2} - V} = \dfrac{\sqrt{2}d(\sqrt{2X^2 - V^2} - V) + \sqrt{2}d(\sqrt{2X^2 - V^2} + V)}{(\sqrt{2X^2 - V^2} + V)(\sqrt{2X^2 - V^2} - V)}$

$= \dfrac{2\sqrt{4X^2 - 2V^2}\,d}{2X^2 - 2V^2} = \dfrac{\sqrt{4X^2 - 2V^2}\,d}{X^2 - V^2}$,

28. Let $\vec{V}_W = a\vec{i} + b\vec{j}$

$\vec{V}_M = -\vec{j}$

$\therefore \vec{V}_{WM} = a\vec{i} + (b + 1)\vec{j}$



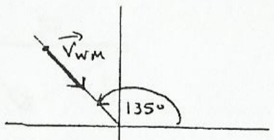
Since this is from the south-west

$\tan 45^\circ = \dfrac{b + 1}{a}$

$\Rightarrow 1 = \dfrac{b + 1}{a} \Rightarrow \boxed{a = b + 1}$ ... result 1 .

$\vec{V}_W = a\vec{i} + b\vec{j}$. $V_M = 3\vec{j}$

$V_{WM} = a\vec{i} + (b - 3)\vec{j}$



Since this is from the north= west

$\tan 135^\circ = \dfrac{b - 3}{a}$

28. contd...

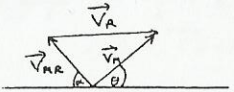
$\Rightarrow -1 = \dfrac{b - 3}{a} \Rightarrow \boxed{-a = b - 3}$ .... result 2

Adding results 1 and 2 gives

$0 = 2b - 2 \Rightarrow b = 1 \quad \therefore a = b + 1 = 1 + 1 = 2$.

Answer: $\vec{V}_W = 2\vec{i} + \vec{j}$.

29.



$\vec{V}_{MR} = -5\cos\alpha\, i + 5\sin\alpha\, j$

$\vec{V}_R = 13\vec{i}$

$\vec{V}_M = (13 - 5\cos\alpha)\vec{i} + 5\sin\alpha\vec{j}$

$\tan\theta = \dfrac{j\text{-component}}{i\text{-component}} = \dfrac{5\sin\alpha}{13 - 5\cos\alpha}$

$\dfrac{d(\tan\theta)}{d\alpha} = \dfrac{(13 - 5\cos\alpha)(5\cos\alpha) - 5\sin\alpha\,(5\sin\alpha)}{(13 - 5\cos\alpha)^2} = 0$

$\Rightarrow 65\cos\alpha - 25\cos^2\alpha - 25\sin^2\alpha = 0$

$\Rightarrow 65\cos\alpha - 25(\cos^2\alpha + \sin^2\alpha) = 0$

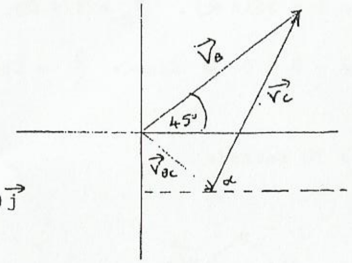
$\Rightarrow 65\cos\alpha - 25 = 0 \Rightarrow \cos\alpha = \dfrac{5}{13}$

The shortest path is where θ is a maximum and therefore where $\tan\theta$ is a maximum, since $\tan\theta$ is an increasing function in θ. That is to say that the shortest path is where $\cos\alpha = 5/13$, and hence $\sin\alpha = 12/13$

In this case, $\vec{V}_M = (13 - 5(\frac{5}{13}))\vec{i} + 5(\frac{12}{13})\vec{j} = \frac{144}{13}\vec{i} + \frac{60}{13}\vec{j}$.

Crossing time $= \dfrac{60}{60/13} = 13$ seconds .

30. (a) $\cos A = \sqrt{1 - \sin^2 A}$



(b)

$\vec{V}_{BC} = 5\vec{i} - 2\vec{j}$

$\vec{V}_C = 5\cos\alpha\vec{i} + 5\sin\alpha\vec{j}$

$\vec{V}_B = (5 + 5\cos\alpha)i + (-2 + 5\sin\alpha)\vec{j}$

This is in a N.E. direction

$\therefore \dfrac{-2 + 5\sin\alpha}{5 + 5\cos\alpha} = \tan 45^\circ = 1$.

30. contd ...

$\Rightarrow -2 + 5 \sin\alpha = 5 + 5 \cos\alpha$

$\Rightarrow -2 + 5 \sin\alpha = 5 + 5\sqrt{1 - \sin^2\alpha}$

$\Rightarrow -7 + 5\sin\alpha = 5\sqrt{1 - \sin^2\alpha}$

$\Rightarrow 49 - 70 \sin\alpha + 25 \sin^2\alpha = 25(1 - \sin^2\alpha)$

$\Rightarrow 50 \sin^2\alpha - 70 \sin\alpha + 24 = 0$

$\Rightarrow 25 \sin^2\alpha - 35 \sin\alpha + 12 = 0$

$\Rightarrow (5 \sin\alpha - 3)(5 \sin\alpha - 4) = 0$

$\Rightarrow \sin\alpha = 3/5$ or $\sin\alpha = 4/5$

$\Rightarrow \cos\alpha = \pm 4/5$ or $\cos\alpha = \pm 3/5$

Possibility 1: $\sin\alpha = 3/5$, $\cos\alpha = 4/5 \Rightarrow \vec{V}_B = 9\vec{i} + 3\vec{j}$. Reject

Possibility 2: $\sin\alpha = 3/5$, $\cos\alpha = -4/5 \Rightarrow \vec{V}_B = \vec{i} + \vec{j}$. Correct

Possibility 3: $\sin\alpha = 4/5$, $\cos\alpha = 3/5 \Rightarrow \vec{V}_B = 8\vec{i} + 2\vec{j}$. Reject

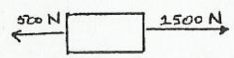
Possibility 4: $\sin\alpha = 4/5$, $\cos\alpha = -3/5 \Rightarrow \vec{V}_B = 2\vec{i} + 2\vec{j}$. Correct

Answer: (i) $\vec{V}_C = -4\vec{i} + 3\vec{j}$ or $-3\vec{i} + 4\vec{j}$

(ii) $\vec{V}_B = \vec{i} + \vec{j}$ or $2\vec{i} + 2\vec{j}$.

EXERCISE 5.A

1. (i)

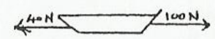


$F = 2500 - 500 = 2000$,

$F = ma = 2000 = 1000a \Rightarrow a = 2$ m/s.

(ii) $\left.\begin{array}{l} a = 2 \\ u = 0 \\ t = 20 \\ s = ? \end{array}\right\}$ $\begin{array}{l} s = ut + \frac{1}{2}at^2 \\ \quad = 0(20) + \frac{1}{2}(2)(400) = 400 \text{ m}. \end{array}$

2.



$F = 100 - 40 = 60$,

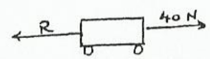
$F = ma \Rightarrow 60 = 120a \Rightarrow a = \frac{1}{2}$ m/s^2.

$F = ma \Rightarrow 60 = 180a \Rightarrow a = \frac{1}{3}$ m/s^2.

3. $F = ma \Rightarrow (t - 40) = 150 (\frac{1}{2}) \Rightarrow T = 115$ N.

$F = ma \Rightarrow (t - 40) = 240 (\frac{1}{2}) \Rightarrow T = 160$ N

4.



$F = ma \Rightarrow (40 - R) = 120 (\frac{1}{8}) \Rightarrow R = 25$ N.

5. $F = ma = -900 = (0.060)\, a \Rightarrow a = 15{,}000$ m/s^2.

$\left.\begin{array}{l} u = 150 \\ v = 0 \\ a = -15{,}000 \\ s = ? \end{array}\right\}$ $\begin{array}{l} v^2 = u^2 + 2as \\ 0 = 22500 + 2(-15{,}000)s \\ \Rightarrow s = 0.75\text{m} = 75\text{cm}. \end{array}$

6. $\left.\begin{array}{l} u = 0 \\ s = 8 \\ t = 8 \\ a = ? \end{array}\right\}$ $\begin{array}{l} s = ut + at^2 \\ \Rightarrow 8 = (0)8 + \frac{1}{2}(a)(64) \\ \Rightarrow a = \frac{1}{4} \text{ m/s}^2. \end{array}$

$F = ma \Rightarrow (T - 20) = 80 (\frac{1}{4}) \Rightarrow T = 40$ N,

7. (i) $\left.\begin{array}{l} u = 0 \\ v = 10 \\ s = 50 \\ a = ? \end{array}\right\}$ $\begin{array}{l} v^2 = u^2 + 2as \\ \Rightarrow 100 = 0 + 2(a)(50) \\ \Rightarrow a = 1 \text{ m/s}^2. \end{array}$

(ii) $F = ma \Rightarrow T - 350 = (800)(1) \Rightarrow T = 1150$ N.

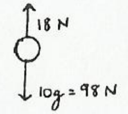
8. (i) $\left.\begin{array}{l} u = 200 \\ v = 0 \\ s = 1 \\ a = ? \end{array}\right\}$ $\begin{array}{l} v^2 = u^2 + 2as \\ \Rightarrow 0 = 40{,}000 + 2(a)(1) \\ \Rightarrow a = -20{,}000. \end{array}$

$F = ma \Rightarrow R = (0.050)(-20{,}000) = -1000$ N.

8. (ii)

$$\left.\begin{array}{l} u = 400 \\ v = 0 \\ a = -20{,}000 \\ s = ? \end{array}\right\} \quad \begin{array}{l} v^2 = u^2 + 2as \\ \Rightarrow 0 = 160{,}000 + 2(-20{,}000)S \\ \Rightarrow S = 4 \text{ m}. \end{array}$$

9.



$$F = 98 - 18 = 80 \text{ N}$$

$$F = ma \Rightarrow 80 = 10(a) \Rightarrow a = 8 \text{ m/s}^2.$$

(i)

$$\left.\begin{array}{l} u = 0 \\ a = 8 \\ t = 10 \\ s = ? \end{array}\right\} \quad \begin{array}{l} S = ut + \frac{1}{2}at^2 \\ \Rightarrow S = 0(10) + \frac{1}{2}(8)(100) = 400 \text{ m}. \end{array}$$

(ii)

$$\left.\begin{array}{l} u = 0 \\ a = 8 \\ t = 20 \\ s = ? \end{array}\right\} \quad \begin{array}{l} S = ut + \frac{1}{2}at^2 \\ \Rightarrow S = (0)(10) + \frac{1}{2}(8)(400) = 1600 \text{ m} \\ \therefore \text{ Distance} = 1600 - 400 = 1200 \text{ m}. \end{array}$$

10.

$$\left.\begin{array}{l} u = 300 \\ v = 200 \\ s = 0.1 \\ a = ? \end{array}\right\} \quad \begin{array}{l} v^2 = u^2 + 2as \\ \Rightarrow 40{,}000 = 90{,}000 + 2(a)(0.1) \\ \Rightarrow a = -250{,}000 \end{array}$$

$$F = ma \Rightarrow -25{,}000 = m(-250{,}000)$$

$$\Rightarrow m = 0.1\text{kg} = 100 \text{ grammes}$$

$$\left.\begin{array}{l} u = 300 \\ v = 0 \\ a = -250{,}000 \\ s = ? \end{array}\right\} \quad \begin{array}{l} v^2 = u^2 + 2as \\ 0 = 90{,}000 + 2(-250{,}000)s \\ \Rightarrow s = 0.18 \text{ m} = 18 \text{ cm} \end{array}$$

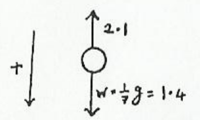
11.

$$\left.\begin{array}{l} u = u \\ v = 0 \\ s = s \\ a = ? \end{array}\right\} \quad \begin{array}{l} v^2 = u^2 + 2as \\ \Longrightarrow 0 = u^2 + 2as \\ \Longrightarrow a = \dfrac{-u^2}{2S} \end{array}$$

$$F = ma \Rightarrow R = m\left(\frac{-u^2}{2S}\right) = \frac{-mu^2}{2S}$$

$$\left.\begin{array}{l} u = 3u \\ s = 5s \\ a = \dfrac{-u^2}{2S} \\ v = ? \end{array}\right\} \quad \begin{array}{l} v^2 = u^2 + 2as \\ \Rightarrow v^2 = 9u^2 + 2\left(\dfrac{-u^2}{2S}\right)(5S) \\ \Rightarrow v^2 = 9u^2 - 5u^2 = 4u^2 \\ \Rightarrow v = 2u \text{ m/s} \end{array}$$

12.



$$F = 1.4 - 2.1 = -0.7 \text{ N}$$

$$F = ma \Rightarrow -0.7 = \left(\tfrac{1}{7}\right)a$$

$$\Rightarrow a = -4.9 \text{ m/s}.$$

12. contd

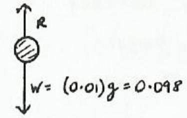
$$\left.\begin{array}{l} u = 1.4 \\ a = -4.9 \\ v = 0 \\ s = ? \end{array}\right\} \quad \begin{array}{l} v^2 = u^2 + 2as \\ \Rightarrow 0 = 1.96 + 2(-4.9)s \\ \Rightarrow s = 0.2 \text{ m} = 20 \text{ cm} \end{array}$$

13. In Air

$$\left.\begin{array}{l} u = 0 \\ a = 9.8 \\ s = 2.5 \\ v = ? \end{array}\right\} \quad \begin{array}{l} v^2 = u^2 + 2as \\ \Rightarrow v^2 = 0 + 2(9.8)(2.5) = 49 \\ \Rightarrow v = 7 \text{ m/s} \end{array}$$

In Material

$$\left.\begin{array}{l} u = 7 \\ v = 0 \\ s = 0.35 \\ a = ? \end{array}\right\} \quad \begin{array}{l} v^2 = u^2 + 2as \\ \Rightarrow 0 = 49 + 2(a)(0.35) \\ \Rightarrow a = -70 \text{ m/s}^2 \end{array}$$



$$F = ma$$

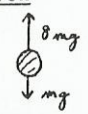
$$\Rightarrow (0.098 - R) = (0.01)(-70)$$

$$\Rightarrow R = 0.798 \text{ N}$$

14. In Air

$$\left.\begin{array}{l} u = 0 \\ s = 4h \\ a = g \\ v = ? \end{array}\right\} \quad \begin{array}{l} v^2 = u^2 + 2as \\ \Longrightarrow v^2 = 0 + 2(g)(4h) \\ \Longrightarrow v = \sqrt{8gh} \end{array}$$

In Marsh



$$F = mg - 8mg = -7mg$$

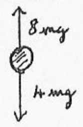
$$F = ma \Rightarrow -7mg = ma \Rightarrow a = -7g$$

$$\left.\begin{array}{l} u = \sqrt{8gh} \\ v = 0 \\ a = -7g \\ s = ? \end{array}\right\} \quad \begin{array}{l} v^2 = u^2 + 2as \\ 0 = 8gh + 2(-7g)s \\ \Rightarrow s = \frac{4}{7}h \end{array}$$

In Air

$$\left.\begin{array}{l} u = 0 \\ s = h \\ a = g \\ v = ? \end{array}\right\} \quad \begin{array}{l} v^2 = u^2 + 2as \\ \Longrightarrow v^2 = 0 + 2gh \\ \Longrightarrow v = \sqrt{2gh} \end{array}$$

IN MARSH



F = 4mg - 8mg = -4 mg

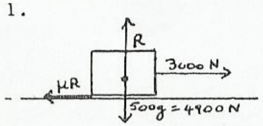
F = ma ⇒ -4 mg = -(4m)a ⇒ a = -g

$$\left.\begin{array}{l} u = \sqrt{2gh} \\ a = -g \\ v = 0 \\ s = ? \end{array}\right\} \quad \begin{array}{l} v^2 = u^2 + 2as \\ 0 = 2gh + 2(-g)s \\ \Rightarrow s = h \end{array}$$

Answer : No.

EXERCISE 5.B
============

1.



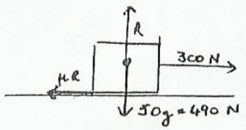
R = 4900.

⇒ μR = (0.4)(4900) = 1960.

F = ma ⇒ (3000 - 1960) = 500a .

⇒ a = 2.08 m/s^2

2.



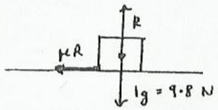
R = 490

⇒ μR = 0.6 (490) = 294 N

F = ma ⇒ (300 - 294) = (50)a

⇒ a = 0.12 m/s^2.

3. (i)



R = 9.8

⇒ Friction = μR = $(\frac{1}{7})$ (9.8)

= 1.4 N

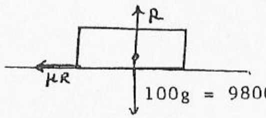
(ii) F = ma

(-1.4) = (1)a ⇒ a = -1.4 m/s^2

The deceleration is 1.4 m/s^2

$$\left.\begin{array}{l} u = 3.5 \\ v = 0 \\ a = -1.4 \\ s = ? \end{array}\right\} \quad \begin{array}{l} v^2 = u^2 + 2as \\ \Rightarrow 0 = 12.25 + 2(-1.4)s \\ \Rightarrow s = 4.375m \end{array}$$

4. (i)



100g = 9800 N

R = 9800 N

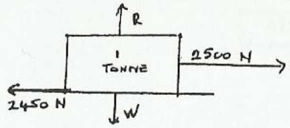
⇒ μR = $\frac{1}{4}$ (9800) = 2450 N

⇒ Limiting friction = 2450 N

4. (ii)

$\frac{2450}{250}$ = 9.8 ∴ 10 slaves needed.

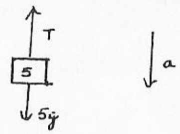
(iii)



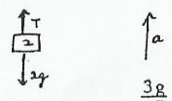
F = ma ⇒ (2500 - 2450) = (1000)a

⇒ a = 0.05 m/s^2

5. (i)



5g - T = 5a



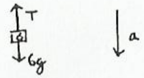
T - 2g = 2a

Solving gives a = $\frac{3g}{7}$ = 4.2 m/s^2

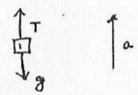
(ii)
$$\left.\begin{array}{l} u = 0 \\ s = 2 \\ a = 4.2 \\ v = ? \end{array}\right\} \quad \begin{array}{l} v^2 = u^2 + 2as \\ \Rightarrow v^2 = 0 + 2(4.2)(2) \\ \Rightarrow v = \sqrt{16.8} \text{ m/s} \end{array}$$

(iii)
$$\left.\begin{array}{l} u = \sqrt{16.8} \\ v = 0 \\ a = -9.8 \\ s = ? \end{array}\right\} \quad \begin{array}{l} v^2 = u^2 + 2as \\ 0 = 16.8 + 2(-9.8)(s) \\ \Rightarrow s = \frac{6}{7} \text{ m} \end{array}$$

6. (i)



6g - T = 6a



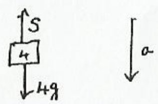
T - g = 1a

Solving gives a = $\frac{5g}{7}$ = 7 m/s^2

(ii) v = u + at = v = 0 + 7(1) = 7 m/s

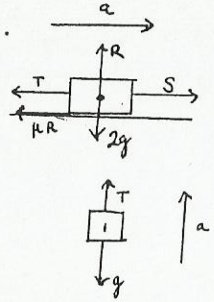
(iii)
$$\left.\begin{array}{l} u = 7 \\ v = 0 \\ a = -9.8 \\ s = ? \end{array}\right\} \quad \begin{array}{l} v^2 = u^2 + 2as \\ \Rightarrow 0 = 49 + 2(-9.8)s \\ \Rightarrow s = 2.5 \text{ m} \end{array}$$

7.



$\boxed{4g - S = 4a}$

7.



$R = 2g \Rightarrow \mu R = \frac{1}{2}R = g$

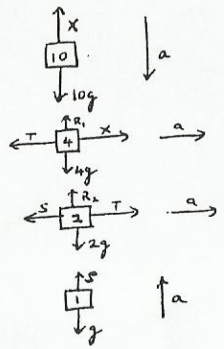
$\boxed{S - T - g = 2a}$

$\boxed{T - g = a}$

Adding gives $a = \frac{2g}{7} = 2.8$ m/s

$\therefore$ $T = 12.6$ N and $S = 28$ N

8. (i)



$\boxed{10g - X = 10a}$

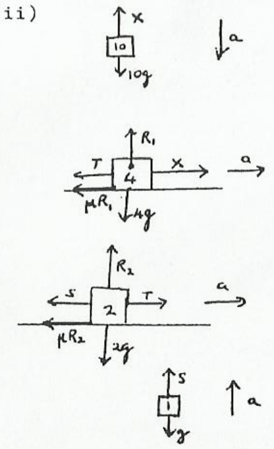
$\boxed{X - T = 4a}$

$\boxed{T - S = 2a}$

$\boxed{S - g = a}$

Adding gives $17a = 9g \Rightarrow a = \frac{9}{17}g$.

(ii)



$\boxed{10g - X = 10a}$

$R_1 = 4g$

$\Rightarrow \mu R_1 = \frac{1}{2}(4g) = 2g$

$\therefore \boxed{X - T - 2g = 4a}$

$R_2 = 2g$

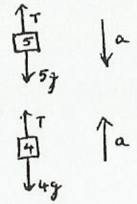
$\mu R_2 = \frac{1}{2}(2g) = g$

$\therefore \boxed{T - S - g = 2a}$

$\boxed{S - g = 1a}$

adding gives $a = \frac{6}{17}g$.

9.



$\boxed{5g - T = 5a}$

$\boxed{T - 4g = 4a}$

Solving gives (i) $a = \frac{1}{9}g$ m/s^2

(ii) $T = \frac{40}{9}g$ N

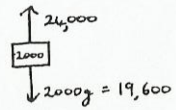
$v = u + at \Rightarrow v = 0 + (\frac{1}{9}g)(2) = \frac{2}{9}g$ m/s

$v^2 = u^2 + 2as \Rightarrow 0 = (\frac{2}{9}g)^2 + 2(-g)s$

$\Rightarrow s = \frac{2}{81}g$ metres

10. (i) As in text.

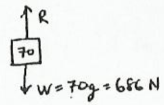
(ii) $1440 + 8(70) = 2000$



$F = ma$

$\Rightarrow (24{,}000 - 19{,}600) = 2000a$

$\Rightarrow a = 2.2$ m/s^2

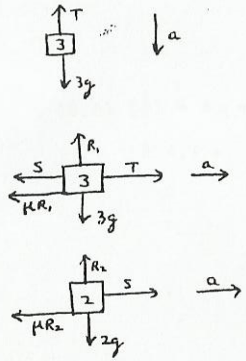


$F = ma$

$\Rightarrow (R - 686) = (70)(2.2)$

$\Rightarrow R = 840$ N

11.



$\boxed{3g - T = 3a}$

$R_1 = 3g$

$\Rightarrow \mu R_1 = \frac{1}{4}(3g) = \frac{3}{4}g$

$\therefore \boxed{T - S - \frac{3}{4}g = 3a}$

$R_2 = 2g$

$\Rightarrow \mu R_2 = \frac{1}{4}(2g) = \frac{1}{2}g$

$\therefore \boxed{S - \frac{1}{2}g = 2a}$

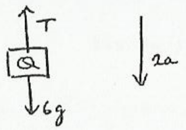
Solving gives $a = \frac{7}{32}g$ m/s^2

$\therefore S = \frac{15}{16}g$ N and $T = \frac{75}{32}g$ N

12. If P moves 1 metre, the string is foreshortened by 2 metres, hence Q falls 2 metres. Hence $y = 2x$.

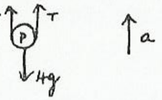
By differentiation, $\frac{d^2y}{dt^2} = 2\frac{d^2x}{dt^2}$

i.e. the acceleration of Q is twice that of P.



$6g - T = (6)(2a)$

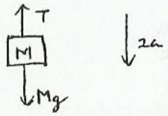
$\Rightarrow \boxed{6g - T = 12a}$



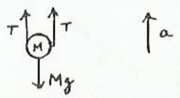
$\boxed{2T - 4g = 4a}$

Solving gives $a = 2.8\ m/s^2$, $T = 25.2$ N

13.



$\boxed{Mg - T = 2Ma}$

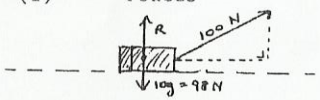


$\boxed{2T - Mg = Ma}$

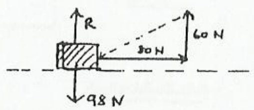
Solving gives $a = \frac{1}{5}g\ m/s^2$: $T = \frac{3}{5}Mg$ N

EXERCISE 5C
============

1. (i) FORCES



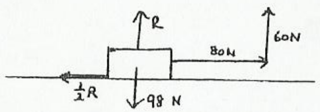
RESOLVED



1. $R + 60 = 98 \Rightarrow R = 38$ N
2. $F = ma \Rightarrow 80 = 10a \Rightarrow a = 8\ m/s^2$

(ii)

FORCES (RESOLVED)



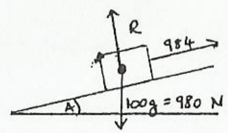
1. (ii) contd...

1. $R + 60 = 98 \Rightarrow R = 38$.
2. $\therefore \frac{1}{2}R = 19$ N = The friction force.

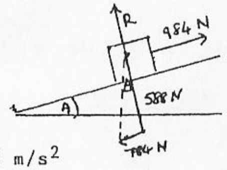
$F = ma \Rightarrow (80-19) = 10a \Rightarrow a = 6.1\ m/s^2$

2. (A) $\tan A = \frac{4}{3} \Rightarrow \sin A = \frac{4}{5}$, $\cos A = \frac{3}{5}$

FORCES

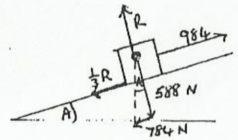


RESOLVED



$F = ma \Rightarrow (984-784) = 100a \Rightarrow a = 2\ m/s^2$

(B) FORCES (RESOLVED)

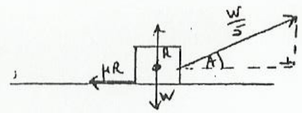


$R = 588 \Rightarrow$ Friction $= \frac{1}{3}R = 196$ N

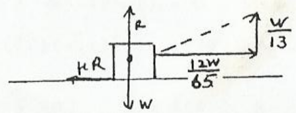
$F = ma \Rightarrow (984-784-196) = 100a$

$\Rightarrow a = 0.04\ m/s^2$

3. FORCES



RESOLVED

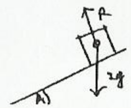


1. $R + \frac{W}{13} = W \Rightarrow R = \frac{12W}{13}$.
2. $\mu R = \frac{12W}{65}$.

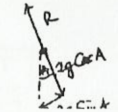
Dividing 2 by 1 gives $\mu = \frac{1}{5}$

4. Since $\sin A = \frac{1}{5}$, $\cos A = \frac{\sqrt{24}}{5}$.

FORCES



RESOLVED

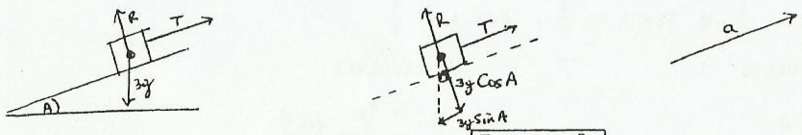


4. (contd ...)

1. $R = 2g \cos A = 2g\left(\frac{\sqrt{24}}{5}\right) = \frac{4\sqrt{6}}{5}g$ Q.E.D.

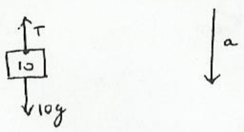
2. $F = ma \Rightarrow 2g \sin A = 2a \Rightarrow a = \frac{1}{5}g$ Q.E.D.

5. 3kg's FORCES — RESOLVED — ACCELERATION



$F = ma \Rightarrow T - 3g \sin A = 3a \Rightarrow \boxed{T - g = 3a}$

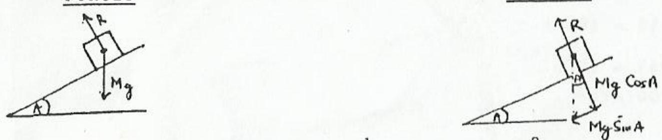
10 kg's FORCES — ACCELERATIONS



$\boxed{10g - T = 10a}$

Solving these gives $a = \frac{9}{13}g$, $T = \frac{40g}{13}$

6. FORCES — RESOLVED



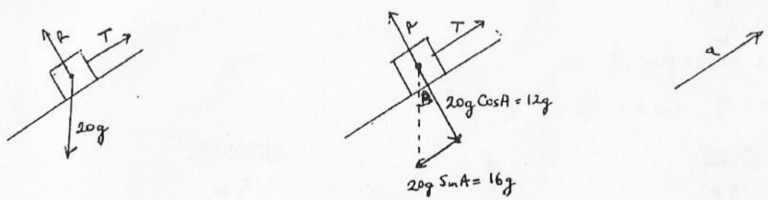
$F = ma \Rightarrow Mg \sin A = Ma \Rightarrow a = \frac{1}{7}g = 1.4\ m/s^2.$

$v = u + at \Rightarrow 7 = 0 + (1.4)t \Rightarrow t = 5s.$

$s = ut + \frac{1}{2}at^2 = 0(5) + \frac{1}{2}(1.4)(25) = 17.5m$

7. $\tan A = \frac{4}{3} \Rightarrow \sin A = \frac{4}{5}$, $\cos A = \frac{3}{5}$.

20kg's FORCES — RESOLVED — ACCELERATION



$\boxed{T - 16g = 20a}$

7. (contd ...) 15kg's FORCES — ACCELERATION

$\boxed{15g - T = 15a}$

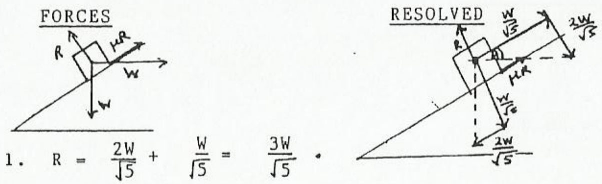
Solving these gives $a = -\frac{1}{35}g$

$\Rightarrow$ acceleration $= \frac{9.8}{35} = 0.28\ m/s^2$

The 15kg rises (since the downward "a" was negative - i.e. it should be upward).

8. $\tan\alpha = 2 \Rightarrow \sin\alpha = \frac{2}{\sqrt{5}}$ $\cos\alpha = \frac{1}{\sqrt{5}}$

FORCES — RESOLVED

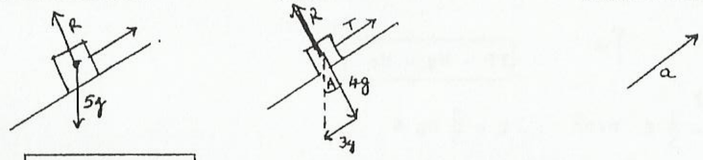


1. $R = \frac{2W}{\sqrt{5}} + \frac{W}{\sqrt{5}} = \frac{3W}{\sqrt{5}}$.

2. $\mu R + \frac{W}{\sqrt{5}} = \frac{2W}{\sqrt{5}} \Rightarrow \mu R = \frac{W}{\sqrt{5}}$.

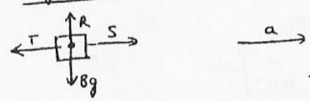
Dividing 1 by 2 $\Rightarrow \mu = \frac{1}{3}$.

9. (i) 5 kg's FORCES — RESOLVED — ACCELERATION



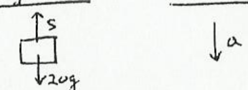
$\boxed{T - 3g = 5a}$

8kg's FORCES — ACCELERATION



$\boxed{S - T = 8a}$

20kg's FORCES — ACCELERATION



$\boxed{20g - S = 20a}$

Adding these gives $17g = 33a \Rightarrow a = \frac{17}{33}g$

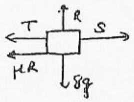
(ii) 5kg's RESOLVED FORCES — ACCELERATION

9. (contd ...)

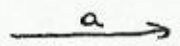
$R = 4g \Rightarrow \mu R = \frac{1}{4}(4g) = g$

$\therefore \; T - g - 3g = 5a \Rightarrow \boxed{T - 4g = 5a}$

8kg's FORCES



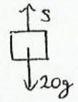
ACCELERATION



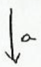
$R = 8g \Rightarrow \mu R = \frac{1}{4}(8g) = 2g$

$\boxed{S - 2g - T = 8a}$

20kg's FORCES



ACCELERATION

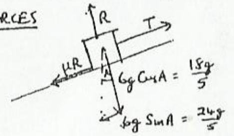


$\boxed{20g - S = 20a}$

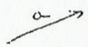
Adding gives : $14g = 33a \Rightarrow a = \frac{14g}{33}$,

10. Since $\tan A = \frac{4}{3}$, $\sin A = \frac{4}{5}$, $\cos A = \frac{3}{5}$

6kg's FORCES



Acceleration

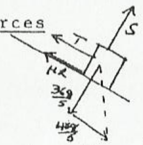


$R = \frac{18g}{5} \Rightarrow \mu R = \frac{1}{6}\left(\frac{18g}{5}\right) = \frac{3g}{5}$

$F = ma \Rightarrow T - \frac{3g}{5} - \frac{24g}{5} = 6a$

$\Rightarrow \boxed{T - \frac{27g}{5} = 6a}$

12kg's Forces



Acceleration

$S = \frac{36g}{5} \Rightarrow \mu S = \frac{1}{6}\left(\frac{36g}{5}\right) = \frac{6g}{5}$

$F = Ma \Rightarrow \frac{48g}{5} - \frac{6g}{5} - T = 12a$

$\Rightarrow \boxed{\frac{42g}{5} - T = 12a}$

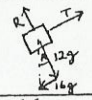
Adding these gives : $3g = 18a$

$\Rightarrow a = \frac{1}{6}g$

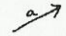
$\Rightarrow T = \frac{32g}{5}$

11. Since $\tan A = \frac{4}{3}$, $\sin A = \frac{4}{5}$, $\cos A = \frac{3}{5}$

(i) 20kg's Forces



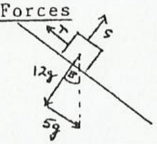
Acceleration



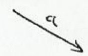
$\boxed{T - 16g = 20a}$

Since $\tan B = \frac{5}{12}$, $\sin B = \frac{5}{13}$, $\cos B = \frac{12}{13}$

13kg's Forces



Acceleration

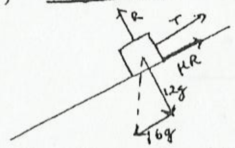


$\boxed{5g - T = 13a}$

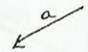
Adding these gives $-11g = 33a \Rightarrow a = -\frac{1}{3}g$ (i.e. they go the other way)

The acceleration of the masses is $\frac{1}{3}g$ m/s^2 and $T = \frac{28}{3}g$ N

(ii) 20kg's FORCES



ACCELERATION



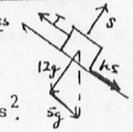
$R = 12g \Rightarrow \mu R = \frac{1}{4}(12g) = 3g$

$16g - 3g - T = 20a \Rightarrow \boxed{13g - T = 20a}$

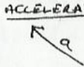
Similarly $S = 12g \Rightarrow \mu S = \frac{1}{4}(12g) = 3g$

$T - 3g - 5g = 13a \Rightarrow \boxed{T - 8g = 13a}$

13kg's FORCES



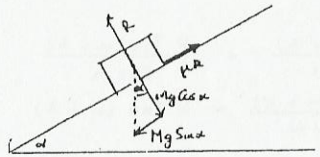
ACCELERATION



Adding these gives $5g = 33a \Rightarrow a = \frac{5}{33}g$ m/s^2.

$\Rightarrow T = \frac{329}{330}g$ N .

12.



1. $Mg\cos\alpha = R$

2. $Mg\sin\alpha - \mu R > 0$ (Since it is moving)

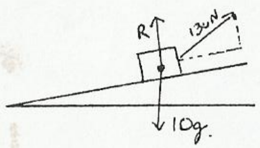
$\Rightarrow Mg\sin\alpha > \mu R$

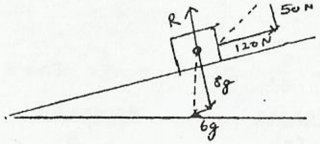
Dividing 2 by 1 $\Rightarrow \tan\alpha > \mu$

i.e. $\tan\alpha > \tan\lambda$

$\Rightarrow \alpha > \lambda$ q.e.d.

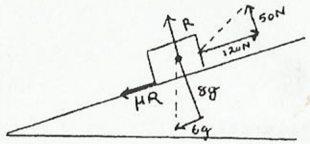
13. (i)





$F = ma \Rightarrow 120 - 6g = 10a \Rightarrow a = 6.12 \ m/s^2$

(ii)



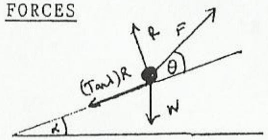
$R + 50 = 8g \Rightarrow R = 28.4 \Rightarrow \mu R = \frac{1}{4}(28.4) = 7.1$

$F = ma \Rightarrow 120 - 6g - 7.1 = 10a$

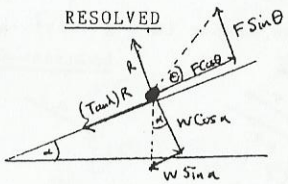
$\Rightarrow a = 5.41 \ m/s^2$

14. $\tan\lambda = \mu$

FORCES



RESOLVED



1. $F\sin\theta + R = W\cos\alpha$

2. $F\cos\theta = (\tan\lambda)R + W\sin\alpha$

$1 \Rightarrow R = W\cos\alpha - F\sin\theta$

$2 \Rightarrow F\cos\theta = \tan\lambda(W\cos\alpha - F\sin\theta) + W\sin\alpha$

$\Rightarrow F(\cos\theta - \tan\lambda\sin\theta)) = W(\cos\alpha\tan\lambda + \sin\alpha)$

Multiply across by $\cos\lambda$.

$\Rightarrow F(\cos\theta\cos\lambda - \sin\lambda\sin\theta) = W(\cos\alpha\sin\lambda + \sin\alpha\cos\lambda)$

$\Rightarrow F\cos(\theta + \lambda) = W\sin(\alpha + \lambda)$

$\Rightarrow F = \dfrac{W\sin(\alpha + \lambda)}{\cos(\theta + \lambda)}$ q.e.d.

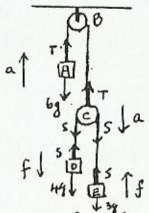
(i) In this case $\theta = 0 \Rightarrow F = \dfrac{W\sin(\alpha + \lambda)}{\cos(-\lambda)} = \dfrac{W\sin(\alpha + \lambda)}{\cos\lambda}$

(ii) In this case $\theta = -\alpha \Rightarrow F = \dfrac{W\sin(\alpha + \lambda)}{\cos(-\alpha-\lambda)} = W\tan(\alpha + \lambda)$

(iii) For $\dfrac{W\sin(\alpha + \lambda)}{\cos(\theta - \lambda)}$ to be minimum, $\cos(\theta - \lambda)$ will be a maximum, therefore $\cos(\theta - \lambda) = 1$ and $F = W\sin(\alpha + \lambda)$.

EXERCISE 5D
============

1.



A : $T - 6g = 6a$

C : $2S - T = 0 \Rightarrow T = 2s$

D : $4g - S = 4(a + f)$

E : $S - 3g = 3(f - a)$

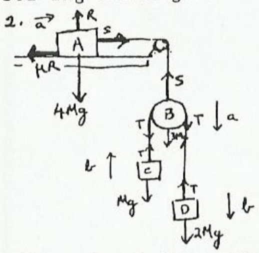
A becomes $2S - 6g = 6a \Rightarrow S = 3a + 3g$.

$\therefore$ D becomes $4g - 3a - 3g = 4a + 4f \Rightarrow 7a + 4f = g$

E becomes $3a + 3g - 3g = 3f - 3a \Rightarrow f - 2a = 0$

Solving these gives $a = \frac{1}{15} g \ m/s^2$, $f = \frac{2}{15} g \ m/s^2$.

2.



Since $R = 4Mg$, $\mu R = \frac{1}{2}(4Mg) = 2Mg$

A : $S - 2Mg = 4Ma$

B : $3Mg + 2T - S = 3Ma$

C : $T - Mg = M(b-a)$

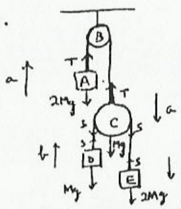
D : $2Mg - T = 2M(b+a)$

Adding C and D $\Rightarrow 3b + a = g$

$A + B - C + D \Rightarrow b + 10a = 4g$

Solving these gives $a = \frac{11}{29} g \ m/s^2$, $b = \frac{6}{29} g \ m/s^2$

3.



A : $T - 2Mg = 2Ma$

C : $Mg + 2S - T = Ma$

D : $S - Mg = M(b-a)$

E : $2Mg - S = 2m(b+a)$

$D + E \Rightarrow Mg = 3Mb + Ma \Rightarrow 3b + a = g$

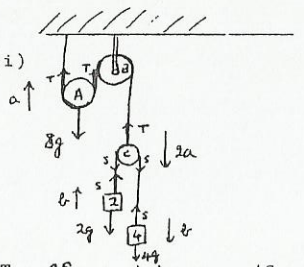
$A + C - D + E \Rightarrow 2Mg = Mb + 6Ma \Rightarrow b + 6a = 2g$

Solving these gives $a = \frac{5}{17}g$ $b = \frac{4}{17} g$

Acceleration of $D = b - a = \frac{-8}{17}$ i.e. $\frac{8}{17}$ downward.

Acceleration of $E = b + a = \frac{9}{17} g$ downward

4. (i)



A : $2T - 8g = 8a$

C : $2s - T = 0(2a)$
$\Rightarrow T = 2s$

D : (2kg): $S - 2g = 2(b - 2a)$

E : (4kg): $4g - S = 4(b + 2a)$

C : $T = 2S \Rightarrow$ A becomes $4S - 8g = 8a \Rightarrow S = 2a + 2g$

$\therefore$ D becomes $2a + 2g - 2g = 2(b-2a) \Rightarrow 6a - 2b = 0$

E becomes $4g - 2a - 2g = 4(b + 2a) \Rightarrow 5a + 2b = g$

Solving these gives $a = \frac{g}{11}$, $b = \frac{3g}{11}$

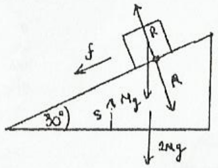
(ii) Pulley A : Acceleration: $a = \frac{g}{11}$ m/s^2

Pulley C : Acceleration: $2a = \frac{2g}{11}$ m/s^2

2kg particle: Acceleration: $b - 2a = g/11$ m/s^2

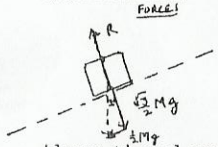
4kg particle: Acceleration: $b + 2a = 5g/11$ m/s^2

5.

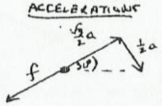


1. The Particle

FORCES



ACCELERATIONS



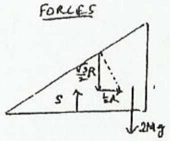
Along the slope : $F = ma \Rightarrow \frac{1}{2}Mg = M(f - \frac{\sqrt{3}}{2}a)$

$\Rightarrow \boxed{g = 2f - \sqrt{3}a}$ 1

Perpendicular to the slope : $F = ma \Rightarrow \frac{\sqrt{3}}{2}Mg - R = M(\frac{1}{2}a)$

$\Rightarrow \boxed{\sqrt{3}\,Mg - 2R = Ma}$ 2

2. The Wedge FORCES



ACCELERATION

a

$F = ma \Rightarrow \frac{1}{2}R = 2\,Ma \Rightarrow \boxed{R = 4Ma}$ 3

5. (contd ...)

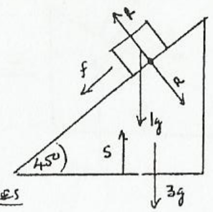
Putting this result into equation 2 gives:

$\sqrt{3}\,Mg - 8\,Ma = Ma$
$\Rightarrow a = \frac{\sqrt{3}}{9}g$ m/s^2

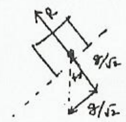
Putting this result into equation 1 gives:

$g = 2f - \sqrt{3}\;(\frac{\sqrt{3}}{9}g) \Rightarrow f = \frac{2}{3}g$ m/s^2

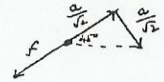
6.



The Particle: FORCES



ACCELERATIONS



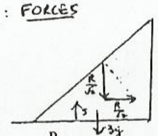
Parallel to the slope:

$F = ma \Rightarrow \frac{g}{\sqrt{2}} = 1\,(f - \frac{a}{\sqrt{2}}) \Rightarrow \boxed{g = \sqrt{2}\,f - a}$ 1

Perpendicular to the slope:

$F = ma \Rightarrow \frac{g}{\sqrt{2}} - R = 1\,(\frac{a}{\sqrt{2}}) \Rightarrow \boxed{g - \sqrt{2}\,R = a}$ 2

The Wedge: FORCES



ACCELERATIONS

a

$F = ma \Rightarrow \frac{R}{\sqrt{2}} = 3a \Rightarrow \boxed{R = 3\sqrt{2}\,a}$ 3

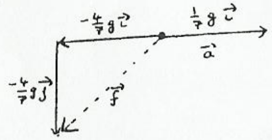
Putting this result into equation 2 gives:

$g - \sqrt{2}\;(3\sqrt{2}a) = a \Rightarrow a = \frac{1}{7}g$ m/s^2

Putting this result into equation 1 gives:

$g = \sqrt{2}f - \frac{1}{7}g \Rightarrow f = \frac{4\sqrt{2}g}{7}$ m/s^2

Here are $\vec{f}$ and $\vec{a}$ resolved:



6. (contd ...)

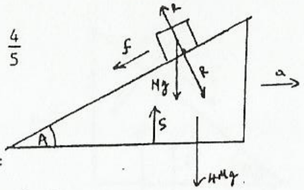
$$\therefore \quad \vec{f} + \vec{a} = \left(-\frac{4}{7} g\vec{i} - \frac{4}{7} g\vec{j}\right) + \frac{1}{7} g\vec{i}$$

$$= -\frac{3}{7} g\vec{i} - \frac{4}{7} g\vec{j}$$

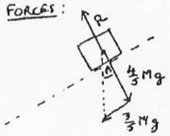
$$\therefore \quad |\vec{f} + \vec{a}| = \sqrt{\frac{9}{49} g^2 + \frac{16}{49} g^2} = \sqrt{\frac{25}{49} g^2} = \frac{5}{7} g$$

This is the magnitude of the actual acceleration of the particle.

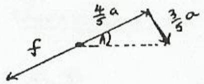
7. Since Tan A = $\frac{3}{4}$, Sin A = $\frac{3}{5}$, Cos A = $\frac{4}{5}$



The Particle: FORCES:



ACCELERATIONS:



Parallel to the slope:

$$F = ma \Rightarrow \frac{3}{5}Mg = M\left(f - \frac{4}{5}a\right)$$

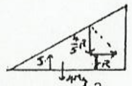
$$\Rightarrow \boxed{3g = 5f - 4a} \quad \ldots\ldots \quad 1$$

Perpendicular to the slope:

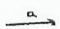
$$F = ma \Rightarrow \frac{4}{5}Mg - R = M\left(\frac{3}{5}a\right)$$

$$\Rightarrow \boxed{4Mg - 5R = 3Ma} \quad \ldots\ldots \quad 2$$

The Wedge: FORCES:



ACCELERATION:



$$F = ma \Rightarrow \frac{3}{5}R = 4Ma \Rightarrow \boxed{R = \frac{20}{3} Ma} \quad \ldots \quad 3$$

Putting this result into equation 2 gives:

$$4Mg - \frac{100}{3} Ma = 3Ma \Rightarrow a = \frac{12g}{109} \text{ m/s}^2$$

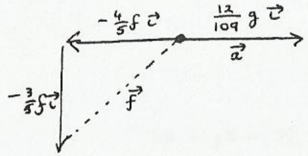
Putting this result into equation 1 gives:

$$3g = 5f - \frac{48}{109} g \Rightarrow 5f = \frac{375}{109} g$$

$$\Rightarrow f = \frac{75}{109} g \text{ m/s}^2$$

Here are f and a resolved:

7. (contd ....)



$$\text{The resultant} = \left(-\frac{4}{5}f + \frac{12}{109} g\right)\vec{i} - \frac{3}{5} f\vec{j}$$

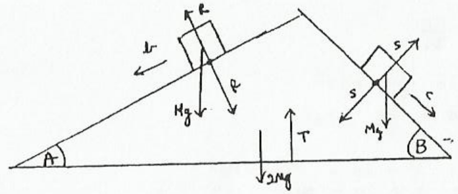
$$= \left(-\frac{60g}{109} + \frac{12}{109} g\right)\vec{i} - \frac{45}{109} g\vec{j}$$

$$= \frac{-48g}{109}\vec{i} - \frac{45}{109} g\vec{j}$$

$$= \frac{3g}{109}\left(-16\vec{i} - 15\vec{j}\right)$$

$$\text{It's magnitude} = \frac{3g}{109}\sqrt{(-16)^2 + (-15)^2} = \frac{3\sqrt{481}}{109} g \text{ m/s}^2$$

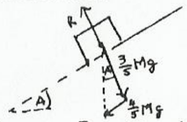
8.



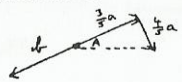
Since Tan a = $\frac{4}{3}$, Sin A = $\frac{4}{5}$, Cos A = $\frac{3}{5}$

Since Tan B = $\frac{3}{4}$, Sin B = $\frac{3}{5}$, Cos B = $\frac{4}{5}$

FIRST PARTICLE: FORCES



ACCELERATIONS



$$\text{Along plane: } F = ma \Rightarrow \frac{4}{5}Mg = M\left(b - \frac{3}{5}a\right)$$

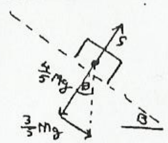
$$\Rightarrow \boxed{4g = 5b - 3a} \quad \ldots \quad 1$$

$$\text{Perpendicular to the plane: } F = ma \Rightarrow \frac{3}{5}Mg - R = M\left(\frac{4}{5}a\right)$$

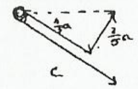
$$\Rightarrow 3Mg - 5R = 4Ma$$

$$\Rightarrow \boxed{R = \frac{3Mg - 4Ma}{5}} \quad \ldots \quad 2$$

OTHER PARTICLE: FORCES



ACCELERATIONS



$$\text{Along the plane: } \frac{3}{5} Mg = M\left(\frac{4}{5}a + c\right)$$

$$\Rightarrow \boxed{3g = 4a + 5c} \quad \ldots\ldots \quad 3$$

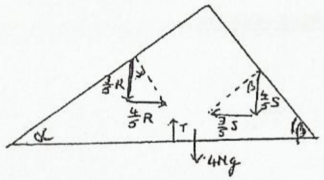
8. contd ....

Perpendicular to the plane : $S - \frac{4}{5}Mg = M(\frac{3}{5}a)$

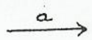
$\Rightarrow$ $5S - 4Mg = 3Ma$

$\Rightarrow$ $\boxed{S = \frac{3Ma + 4Mg}{5}}$ 4

The Wedge: FORCES



ACCELERATION



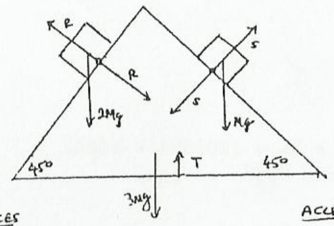
$\frac{4R}{5} - \frac{3}{5}S = 2Ma$.

$\Rightarrow \frac{4}{5}\left(\frac{3Mg - 4Ma}{5}\right) - \frac{3}{5}\left(\frac{3Ma + 4Mg}{5}\right) = 2Ma$. (from 2 and 4)

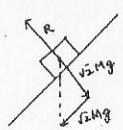
$\Rightarrow \frac{1}{25}(12Mg - 16Ma - 9Ma - 12Mg) = 2Ma$.

$\Rightarrow a = 0$.

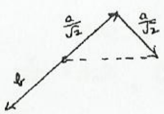
9.



The 2M mass: FORCES



ACCELERATIONS



Along the plane: $\sqrt{2}\ Mg = 2M(b - \frac{a}{\sqrt{2}})$

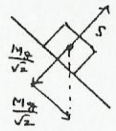
$\Rightarrow$ $\boxed{g = \sqrt{2}\ b - a}$ 1

Perpendicular to the plane: $\sqrt{2}\ Mg - R = 2M\ (\frac{a}{\sqrt{2}})$

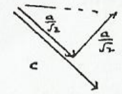
$\Rightarrow$ $\boxed{R = \sqrt{2}Mg - \sqrt{2}\ Ma}$ 2

9. contd ....

The M mass: FORCES



ACCELERATIONS



Along the plane: $\frac{Mg}{\sqrt{2}} = M\ (c + \frac{a}{\sqrt{2}})$

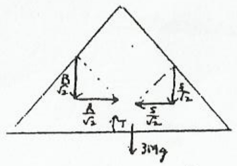
$\Rightarrow$ $\boxed{g = \sqrt{2}\ c + a}$ 3

Perpendicular to the plane:

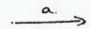
$S - \frac{Mg}{\sqrt{2}} = M\ (\frac{a}{\sqrt{2}})$

$\Rightarrow$ $\boxed{S = \frac{Ma}{\sqrt{2}} + \frac{Mg}{\sqrt{2}}}$ 4

The Wedge: FORCES



ACCELERATION



Along the horizontal: $\frac{R}{\sqrt{2}} - \frac{S}{\sqrt{2}} = 3Ma$

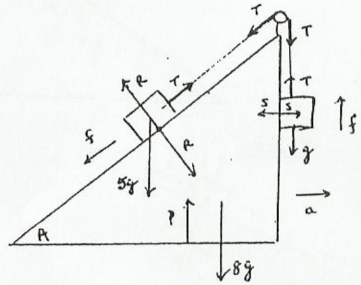
$\Rightarrow R - S = 3\sqrt{2}\ Ma$

$\Rightarrow \sqrt{2}\ Mg - \sqrt{2}\ Ma - \frac{Ma}{\sqrt{2}} - \frac{Mg}{\sqrt{2}} = 3\sqrt{2}Ma$ (from 2 and 4)

$\Rightarrow 2\ Mg - 2\ Ma - Ma - Mg = 6Ma$

$\Rightarrow a = \frac{1}{9}g\ m/s^2$

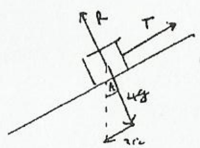
10.



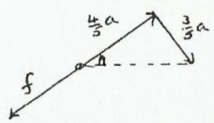
Since Tan A = $\frac{3}{4}$, Sin A = $\frac{3}{5}$, Cos A = $\frac{4}{5}$

The 5kg Mass:

Forces



Acceleration



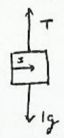
10. (contd ...)

1. Along the plane: $3g - T = 5\left(f - \frac{4}{5}a\right)$

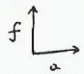
2. Perpendicular to the plane
$4g - R = 5\left(\frac{3}{5}a\right)$

The 1kg Mass:

Forces



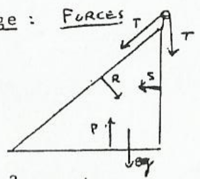
Accelerations

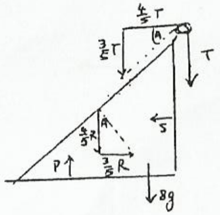


3. Along the vertical : $T - g = f$

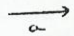
4. Along the horizontal: $S = a$.

The Wedge : Forces



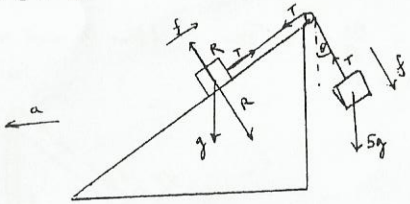


Acceleration



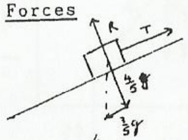
5. $\frac{3}{5}R - \frac{4}{5}T - S = 8a$

11.

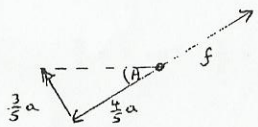


The 1kg Mass:

Forces



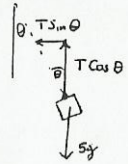
Acceleration



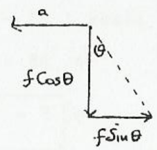
1. $T - \frac{3}{5}g = 1\left(f - \frac{4}{5}a\right)$
2. $R - \frac{4}{5}g = 1\left(\frac{3}{5}a\right)$

11. contd ...

The 5kg Mass: Forces



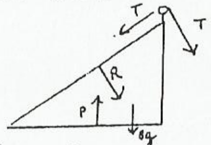
Accelerations



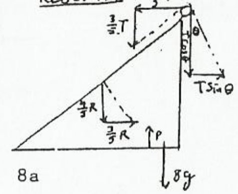
3. $5g - T\cos\theta = 5(f\cos\theta)$

4. $T\sin\theta = 5(a - f\sin\theta)$

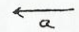
The Wedge : Forces



Resolved



Acceleration



5. $\frac{4}{5}T - \frac{3}{5}R - T\sin\theta = 8a$

CHAPTER 6. WORK, ENERGY AND GRAVITATION.

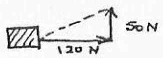
EXERCISE 6.A

1. Work = Fs = 80(30) = 2400 Joules.

   Power = $\frac{\text{Work}}{\text{Time}} = \frac{2400}{10}$ = 240 Watts.

2. Work = Fs = 200(12) = 2400J.

   Power = $\frac{2400}{8}$ = 300W.

3. 

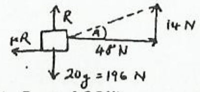
   Work = Fs = 120(50) = 6000J

   Power = $\frac{6000}{60}$ = 100W

4. Power = Fv = 600(20) = 12000W

5. P = Fv ⇒ 100,000 = F(5) ⇒ F = 20,000N

6. Since $\tan A = \frac{7}{24}$, $\cos A = \frac{24}{25}$, $\sin A = \frac{7}{25}$.



(i) $R + 14 = 196 \Rightarrow R = 182N$

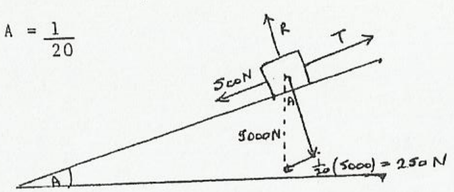
(ii) $\mu R = \frac{1}{7}(182) = 26N$

(iii) Work = Fs = 48(30) = 1440J

(iv) $F = ma \Rightarrow (48-26) = 20a \Rightarrow a = 1.1\ m/s^2$

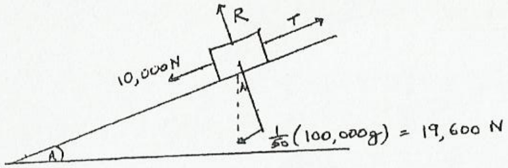
(v) Power = $\frac{\text{work}}{\text{time}} = \frac{1440}{20}$ = 72W

7. Sin A = $\frac{1}{20}$



No acceleration $\Rightarrow$ T = 500 + 250 = 750 N
Power = Fv = 750 (12) = 9000W

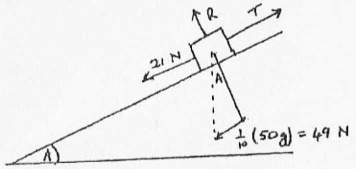
8.



No acceleration $\Rightarrow$ T = 19,600 + 10,000 = 29,600 N .

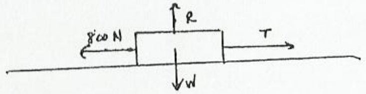
P = Fv $\Rightarrow$ P = (29,600)(10) = 296,000W
= 296 kW.

9.



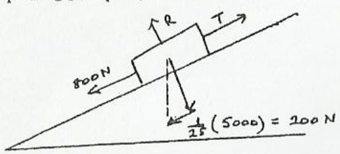
No acceleration $\Rightarrow$ T = 21 + 49 = 70 N .
P = Fv $\Rightarrow$ 350 = (70)v $\Rightarrow$ v = 5 m/s .

10. (i)



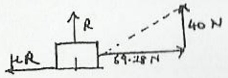
T = 800N .
P = Fv $\Rightarrow$ P = 800 (50) = 40,000W = 40 kW.

(ii)



T = 800 + 200 = 1000
P = Fv $\Rightarrow$ 40,000 = (1000)v $\Rightarrow$ v = 40 m/s.

11. (i) Horizontal = T Cos θ = 80 (0.8660) = 69.28 N
Vertical = T Sin θ = 80(0.5) = 40N



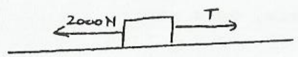
11.
(ii) R + 40 = 196 $\Rightarrow$ R = 156 N

(iii) $\mu R = \frac{1}{3}(156) = 52$ N

F = ma $\Rightarrow$ (69.28 − 52) = 20a $\Rightarrow$ a = 0.864 m/s

(iv) Work = Fs = 69.28(0.5) = 34.64J

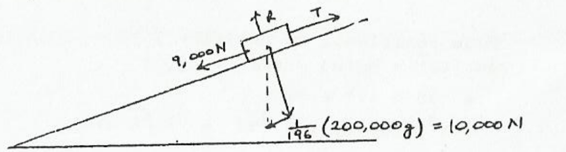
12.



P = Tv $\Rightarrow$ 500,000 = T(10) $\Rightarrow$ T = 50,000

F = ma $\Rightarrow$ (50,000 − 2,000) = 200,000a

$\Rightarrow$ a = 0.24 m/s^2



P = Tv $\Rightarrow$ 500,000 = T(20) $\Rightarrow$ T = 25,000 N

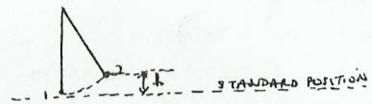
F = ma $\Rightarrow$ (25,000 − 10,000 − 9,000) = 200,000a

$\Rightarrow$ a = 0.03 m/s^2

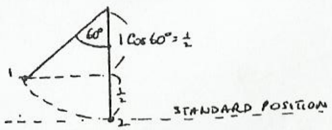
EXERCISE 6B.

1. $\frac{1}{2}M(20)^2 + Mg(0) = \frac{1}{2}Mv^2 + Mg\ (10)$
$\Rightarrow$ 200 = $\frac{1}{2}v^2$ + 98
$\Rightarrow$ v = $\sqrt{204}$ = 14.28 m/s.

2. $\frac{1}{2}M(2)^2 + Mg(0) = \frac{1}{2}M(0)^2 + Mgh$
$\Rightarrow$ 2 = 9.8h
$\Rightarrow$ h = $\frac{2}{9.8}$ = $\frac{10}{49}$ m



3.

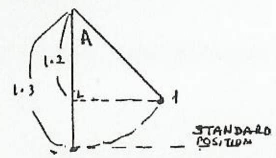


$\frac{1}{2}M(0) + Mg(\frac{1}{2}) = \frac{1}{2}Mv^2 + Mg(0)$
$\Rightarrow$ v^2 = g $\Rightarrow$ v = $\sqrt{g}$ m/s.

4. Work = $\frac{1}{2}Mv_2^2 - \frac{1}{2}Mv_1^2$
= $\frac{1}{2}(1000)(500)^2 - \frac{1}{2}(1000)(300)^2$
= 125,000,000 − 45,000,000 = 80,000,000J
= 80,000 kJ

Power = $\frac{80,000}{20}$ = 4,000 kW

5.



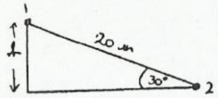
$\frac{1}{2}M(0) + Mg(0.1) = \frac{1}{2}Mv^2 + Mg(0)$
$\Rightarrow$ v^2 = 0.2g $\Rightarrow$ v = $\sqrt{0.2g}$, = $\sqrt{\frac{g}{5}}$

6. Work done = Energy gained = Mgh = (3000)(9.8)(20)
= 588,000J = 588 kJ

7. (i) Work done per minute = Mgh = (100)(9.8)(30) = 294,000J
= 294kJ

(ii) Power = $\frac{\text{work}}{\text{time}} = \frac{294}{60} = 4.9\text{kW}$

8. Since no force, apart from the gravitational force, does work on the sleigh.

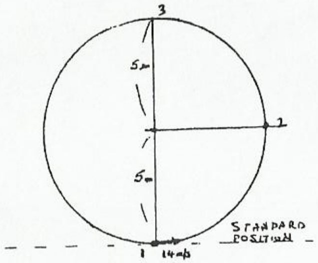


$h = 20 \sin 30^\circ = 20(\frac{1}{2}) = 10\text{m}$

$\frac{1}{2}M(0)^2 + Mg(10) = \frac{1}{2}Mv^2 + Mg(0)$

$\Rightarrow v^2 = 20g = 196 \Rightarrow v = 14$ m/s.

9.



From position 1 to position 2

$\frac{1}{2}M(14)^2 + Mg(0) = \frac{1}{2}Mv^2 + Mg(5)$

$\Rightarrow 98 = \frac{1}{2}v^2 + 49$

$\Rightarrow v^2 = 98 \Rightarrow v = \sqrt{98} = 7\sqrt{2}$ m/s.

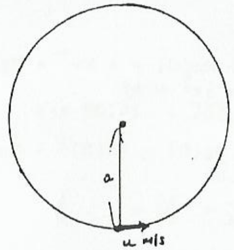
From position 1 to position 3

$\frac{1}{2}M(14)^2 + Mg(0) = \frac{1}{2}Mv^2 + Mg(10)$

$\Rightarrow 98 = \frac{1}{2}v^2 + 98$

$\Rightarrow v = 0$ It will just reach (3)

10. (i) K.E. + P.E. $= \frac{1}{2}M(2ga) + Mg(0) = Mga$



(ii) $Mga = \frac{1}{2}M(0)^2 + Mgh$

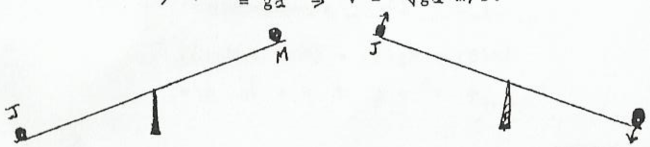
$\Rightarrow h = a$

Ans: a metres

(iii) $Mga = \frac{1}{2}Mv^2 + Mg(\frac{1}{2}a)$

$\Rightarrow v^2 = ga \Rightarrow v = \sqrt{ga}$ m/s.

11.



$$\underbrace{\frac{1}{2}M(0)^2 + Mg(0)}_{\text{Johnny}} + \underbrace{\frac{1}{2}(\frac{3}{2}M)(0)^2 + \frac{3}{2}Mg(\frac{1}{2})}_{\text{Margery}} = \underbrace{\frac{1}{2}Mv^2 + Mg(\frac{1}{2})}_{\text{Johnny}} + \underbrace{\frac{1}{2}(\frac{3}{2}M)v^2 + \frac{3}{2}Mg(0)}_{\text{Margery}}$$

Energy Before = Energy After

$\Rightarrow \frac{3}{4}Mg = \frac{1}{2}Mv^2 + \frac{1}{2}Mg + \frac{3}{4}Mv^2$

$\Rightarrow Mg = 5Mv^2 \Rightarrow v = \sqrt{\frac{g}{5}} = \sqrt{\frac{9.8}{5}} = \sqrt{1.96} = 1.4$ m/s.

EXERCISE 6.C

1. $F = ma \Rightarrow Ma = \frac{G M M_m}{r^2}$

$\Rightarrow a = \frac{G M_m}{r^2} = \frac{6.7 \times 10^{-11} \times 7.4 \times 10^{22}}{(1.7)^2 \times 10^{12}}$

$= \frac{49.58 \times 10^{11}}{2.89 \times 10^{12}} = 1.7 \text{ m/s}^2$

2. (i) $6.4 \times 10^6 + 1.6 \times 10^6 = 8 \times 10^6$

(ii) $a = \frac{G M_e}{r^2} = \frac{6.7 \times 10^{-11} \times 6 \times 10^{24}}{8^2 \times 10^{12}} = 6.28 \text{ m/s}^2$

3. (a) $F_p : F_q = \frac{1}{r_p^2} : \frac{1}{r_q^2} = \frac{1}{1^2} : \frac{1}{2^2} = \frac{1}{1} : \frac{1}{4} = 4 : 1$

(b) $F_p : F_R = \frac{1}{r_p^2} = \frac{1}{r_R^2} = \frac{1}{1^2} : \frac{1}{3^2} = 9 : 1$

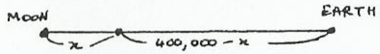
4. $a = \frac{G M_p}{r^2}$

$\Rightarrow 4.44 = \frac{6.66 \times 10^{-11} \times 6 \times 10^{23}}{r^2}$

$\Rightarrow r^2 = \frac{6.66 \times 6 \times 10^{12}}{4.44} = 9 \times 10^{12}$.

$\Rightarrow r = 3 \times 10^6$.

5.



Let M = Mass of the spacecraft

$F_m = \frac{G M_1 M_2}{r^2} = \frac{G M M}{x^2}$

$F_e = \frac{G M_1 M_2}{r^2} = \frac{G M (81M)}{(400,000 - x)^2}$

$F_m = F_e \Rightarrow \frac{G M M}{x^2} = \frac{G M (81M)}{(400,000 - x)^2}$

$\Rightarrow (400,000 - x)^2 = 81x^2$

$\Rightarrow 400,000 - x = 9x$

$\Rightarrow x = 40,000$ km

$\therefore$ Distance from earth = 360,000 km.

7. $F = \frac{G M_1 M_2}{r^2}$ where r = 4 x 10^5 km = 4 x 10^8 m.

$= \frac{6.7 \times 10^{-11} \times 1000 \times 7.4 \times 10^{22}}{(4 \times 10^8)^2}$

$= \frac{49.58 \times 10^{14}}{16 \times 10^{16}} = 0.031$ **N**.

8. Let M = Mass of the baby

$F_J : F_G = \frac{G M (1.8 \times 10^{27})}{(6 \times 10^8)^2} : \frac{G M (100)^2}{4}$

$5 \times 10^{9} : 2500$

The Jupiter-force is much greater
The claim is, therefore, false.

CHAPTER 7. IMPACTS AND COLLISIONS.

Exercise 7.A

1. $I = Mv - Mu = (0.125)(40) - (0.125)(0) = 5$ Ns
2. $I = Mv - Mu = (\tfrac{1}{4})(20) - \tfrac{1}{4}(-40) = 15$ Ns.
3. $M_1 U_1 + M_2 U_2 = M_1 V_1 + M_2 V_2$
(Hammer) . (Stake) (Hammer) (Stake)
$\Rightarrow 4(2) + 1(0) = 4(0) + 1(V_2) \Rightarrow V_2 = 8$ m/s
(i) $\vec{I} = M\vec{v} - M\vec{u} = (1)(-8\vec{j}) - (1)(0) = -8\vec{j}$ Ns
(ii) $\vec{I} = M\vec{v} - M\vec{u} = 4(\vec{0}) - 4(-2\vec{j}) = 8\vec{j}$ Ns.
4. $I = |M\vec{v} - M\vec{u}| = |(0.1)(0) - (0.1)(8)| = 0.8$ Ns
5. $I = |M\vec{v} - M\vec{u}| = |2(0) - 2(5)| = 10$ Ns
6. $M_1 U_1 + M_2 U_2 = M_1 V_1 + M_2 V_2$
$\Rightarrow (0.15)(200) + (3)(0) = (0.15)(100) + (3)V_2 \Rightarrow V_2 = 5$ m/s
7. $M_1\vec{u}_1 + M_2\vec{u}_2 = (M_1 + M_2)\vec{v}$
$(0.1)\ 400\vec{i} + (3)(10\vec{j}) = (0.1 + 3)\vec{v}$
$\Rightarrow 3.1\vec{v} = 40\vec{i} + 30\vec{j}$
$\Rightarrow \vec{v} = \frac{10}{31}(40\vec{i} + 30\vec{j}) \Rightarrow |\vec{v}| = \frac{10}{31}\sqrt{40^2 + 30^2} = \frac{500}{31}$ m/s

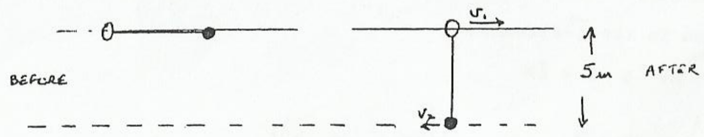
8. $M_1 u_1 + M_2 u_2 = (M_1 + M_2)v$
$(0.1)(200) + M_2(0) = (0.1 + M_2)\,10$
$\Rightarrow 20 = 1 + 10 M_2 = M_2 \Rightarrow 1.9$ kg

9. $M_1 U_1 + M_2 U_2 = (M_1 + M_2)V$
$(0.05)U_1 + (1.45)(0) = (0.05 + 1.45)(4)$
$\Rightarrow 0.05U_1 = 6$
$\Rightarrow U_1 = 120$ m/s

10. Step 1: Find the speed of the joint mass after impact.
$\tfrac{1}{2}MV_1^2 + Mgh_1 = \tfrac{1}{2}MV_2^2 + Mgh_2$
$\tfrac{1}{2}(3)V_1^2 + (3)g(0) = \tfrac{1}{2}(3)(0)^2 + (3)(9.8)(10)$, since it rises 10m.
$\Rightarrow V_1^2 = 196 \Rightarrow V_1 = 14$ m/s

Step 2: To find speed of the bullet before impact:
$M_1 U_1 + M_2 U_2 = (M_1 + M_2)V$
$(0.1)U_1 + (2.9)(0) = (0.1 + 2.9)(14)$
$\Rightarrow U_1 = 420$ m/s

11.



Conservation of momentum $\Rightarrow M(0) + M(0) = Mv_1 - Mv_2$
$\Rightarrow v_1 = v_2$

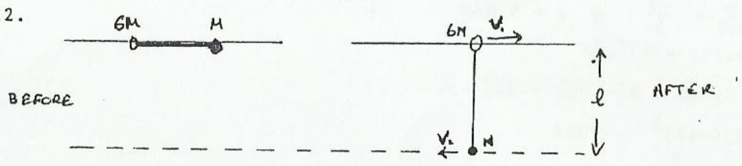
Conservation of Energy $\Rightarrow$

$\underbrace{Mg(5) + \tfrac{1}{2}M(0)^2}_{\text{RING}} + \underbrace{Mg(5) + \tfrac{1}{2}M(0)^2}_{\text{BEAD}} = \underbrace{Mg(5) + \tfrac{1}{2}Mv_1^2}_{\text{RING}} + \underbrace{Mg(0) + \tfrac{1}{2}Mv_2^2}_{\text{BEAD}}$

$\Rightarrow v_1^2 + v_2^2 = 10g$
But $v_1 = v_2$; therefore $v_1^2 + v_1^2 = 10g \Rightarrow v_1 = \sqrt{5g} = 7$ m/s.

Answer : Both speeds are 7 m/s.

12.



Conservation of momentum $\Rightarrow 6M(0) + M(0) = 6MV_1 - MV_2$
$\Rightarrow V_2 = 6V_1$

12. contd ...

Conservation of Energy =

$6Mg\ell + \frac{1}{2}(6M)(0)^2 + Mg\ell + \frac{1}{2}M(0)^2 = 6Mg\ell + \frac{1}{2}(6M)V_1^2 + Mg(0) + \frac{1}{2}Mv^2$

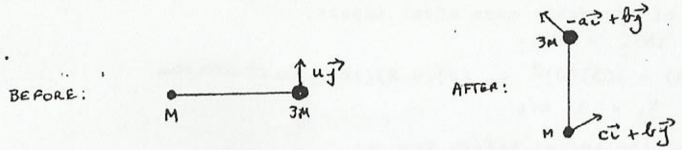
$\Rightarrow \quad 3V_1^2 + \frac{1}{2}V_2^2 = g\ell.$

But $V_2 = 6V_1$: therefore $3V_1^2 + \frac{1}{2}(6V_1)^2 = g\ell$

$\Rightarrow 21V_1^2 = gl \Rightarrow V_1 = \sqrt{\frac{gl}{21}}$

$V_2 = 6V_1 = 6\sqrt{\frac{gl}{21}} = \sqrt{\frac{12gl}{7}}$

13.



We are taking a, b, c to be <u>positive</u> constants.

Conservation of momentum in the $\vec{i}$ - direction:

$3M(0) + M(0) = 3M(-a) + Mc \Rightarrow c = 3a$

Conservation of momentum in the $\vec{j}$-direction :

$3Mu + M(0) = 3Mb + Mb \Rightarrow b = \frac{3}{4}u$

Conservation of Energy:

$\frac{1}{2}(3M)u^2 + \frac{1}{2}M(0)^2 = \frac{1}{2}(3M)(a^2 + b^2) + \frac{1}{2}(M)(c^2 + b^2)$

$\Rightarrow 3a^2 + 4b^2 + c^2 = 3u^2$

Putting $b = \frac{3}{4}u$ and $c = 3a$, this reads:

$3a^2 + 4\left(\frac{9u^2}{16}\right) + 9a^2 = 3u^2$

$\Rightarrow 12a^2 = \frac{3}{4}u^2 \Rightarrow a^2 = \frac{1}{16}u^2 \Rightarrow a = \frac{1}{4}u.$

$b = \frac{3}{4}u$ and $c = 3a = \frac{3}{4}u.$

The 3M's velocity $= -a\vec{i} + b\vec{j} = -\frac{1}{4}u\vec{i} + \frac{3}{4}u\vec{j}$

The M's velocity $= c\vec{i} + b\vec{j} = \frac{3}{4}u\vec{i} + \frac{3}{4}u\vec{j}.$

EXERCISE 7.B

1. (i) $\frac{NEW}{OLD} = -e \Rightarrow \frac{p}{-10} = \frac{-3}{5} \Rightarrow p = 6$ m/s

The new velocity $= 6\vec{j}$ m/s

(ii) $\vec{I} = m\vec{v} - m\vec{u} = 2(6\vec{j}) - 2(-10\vec{j}) = 32\vec{j}$ Ns.

(iii) $KE_{before} = \frac{1}{2}(2)(-10)^2 = 100J$,

$KE_{after} = \frac{1}{2}(2)(6)^2 = 36J$

Loss = 100 - 36 = 64J

2. (i) To find speed at impact:

$v^2 = u^2 + 2as \Rightarrow v^2 = 0^2 + 2(9.8)(2.5) \Rightarrow v = 7$ m/s.

(ii) $\frac{NEW}{OLD} = -e \Rightarrow \frac{p}{-7} = -\frac{4}{7} \Rightarrow p = 4.$ Ans: 4m/s.

(iii) $\vec{I} = M\vec{v} - M\vec{u} = (1)(4\vec{j}) - (1)(-7\vec{j}) = 11\vec{j}$ Ns.

(iv) Loss $= \frac{1}{2}Mu^2 - \frac{1}{2}Mv^2 = \frac{1}{2}(1)(49) - \frac{1}{2}(1)(16) = 16\frac{1}{2}J$

3. (i) $v^2 = u^2 + 2as \Rightarrow v^2 = 0^2 + 2(9.8)(10) \Rightarrow v = 14$ m/s

(ii) $\vec{I} = M\vec{v} - M\vec{u} = (6)(\vec{0}) - (6)(-14\vec{j}) = 84\vec{j}$ Ns.

(iii) Loss $= \frac{1}{2}Mu^2 - \frac{1}{2}Mv^2 = \frac{1}{2}(6)(14)^2 - \frac{1}{2}(6)(0)^2 = 588J$

4. (i)

Before	(Mass)	After
$5\vec{i} - 8\vec{j}$	2kg	$5\vec{i} + p\vec{j}$

$\frac{NEW}{OLD} = -e \Rightarrow \frac{p}{-8} = -\frac{3}{4} \Rightarrow p = 6.$ Ans: $5\vec{i} + 6\vec{j}$.

(ii) $\frac{1}{2}Mu^2 - \frac{1}{2}Mv^2 = \frac{1}{2}(2)(25+64) - \frac{1}{2}(2)(25+36) = 28$ J.

(iii) $\vec{I} = M\vec{v} - M\vec{u} = 2(5\vec{i}+6\vec{j}) - 2(5\vec{i}-8\vec{j}) = 28\vec{j}$Ns.

5. (i) $\frac{NEW}{OLD} = -e \Rightarrow \frac{4}{-6} = -e \Rightarrow e = \frac{2}{3}$

(ii) $KE_{before} = \frac{1}{2}(4)(64 + 36) = 200J$

$KE_{after} = \frac{1}{2}(4)(64 + 16) = 160J$

% Loss $= \frac{40}{200} \times \frac{100}{1} = 20\%$

(iii) $\text{Tan } A = \frac{6}{8} = \frac{3}{4}$, $\text{Tan } B = \frac{4}{8} = \frac{1}{2}$

$\frac{\text{Tan } B}{\text{Tan } A} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$

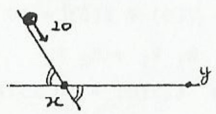
6.

Before	(Mass)	After
$u\text{Cos}A\vec{i} + u\text{Sin}A\vec{j}$	M	$v\text{Cos}B\vec{i} + v\text{Sin}B\vec{j}$

1. $u\text{Cos}A = v\text{Cos}B$ (i - velocity remains the same)
2. $\frac{NEW}{OLD} = -e \Rightarrow \frac{v\text{Sin}B}{-u\text{Sin}A} = -e \Rightarrow eu\text{Sin } A = v\text{Sin}B$

Dividing 2 by 1 gives : $e \text{ Tan } A = \text{Tan } B \Rightarrow e = \frac{\text{Tan } B}{\text{Tan } A}$

7. (i) $\vec{u} = 20(\frac{3}{5})\vec{i} - 20(\frac{4}{5})\vec{j} = 12\vec{i} - 16\vec{j}$



Before	Mass	After
$12\vec{i} - 16\vec{j}$	M	$12\vec{i} + p\vec{j}$

$\frac{NEW}{OLD} = -e \Rightarrow \frac{p}{-16} = \frac{-3}{4} \Rightarrow p = 12.$

7. contd ....

New Velocity = $12\vec{i} + 12\vec{j}$

New Speed = $\sqrt{144 + 144} = 12\sqrt{2}$ m/s

(ii) $\vec{I} = M\vec{v} - M\vec{u} = M(12\vec{i} + 12\vec{j}) - M(12\vec{i} - 16\vec{j}) = 28M\vec{j}$ Ns.

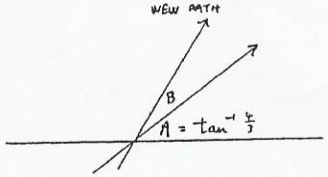
Magnitude = 28M Ns.

iii) $\frac{1}{2}Mu^2 - \frac{1}{2}Mv^2 = \frac{1}{2}M(144 + 256) - \frac{1}{2}M(144 + 144) = 56M$ J.

(iv) $\tan B = \frac{12}{12} = 1 \Rightarrow B = 45^\circ$.

$\tan(A+B) = \pm \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$= \pm \dfrac{\frac{4}{3} + 1}{1 - (\frac{4}{3})(1)} = \pm 7.$



8. Speed at impact : $v^2 = u^2 + 2as \Rightarrow v^2 = 0^2 + 2gh \Rightarrow v = \sqrt{2gh}$

Speed after impact = $\frac{3}{4}\sqrt{2gh}$

New Height : $v^2 = u^2 + 2as \Rightarrow 0^2 = \frac{9}{16}(2gh) + 2(-g)s$

$\Rightarrow s = \frac{9}{16}h$

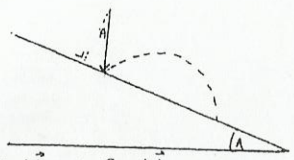
Each new height = $\frac{9}{16}$ ths of the previous height.

$\therefore$ Total distance = $h + 2(\frac{9}{16}h) + 2(\frac{9}{16} \times \frac{9}{16}h) + ---$

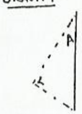
$= h + (S_\infty$ of a G.P. with $a = \frac{9}{8}h$, $r = \frac{9}{16}$)

$= h + \dfrac{9h/8}{1 - \frac{9}{16}} = h + \frac{18h}{7} = \frac{25h}{7}$.

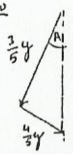
9.



GRAVITY



RESOLVED



$\vec{u} = u\sin A\vec{i} - u\cos A\vec{j}$

$= 20(\frac{4}{5})\vec{i} - 20(\frac{3}{5})\vec{j} = 16\vec{i} - 12\vec{j}$.

Before	(Mass)	After
$16\vec{i} - 12\vec{j}$	M	$16\vec{i} + p\vec{j}$

$\frac{NEW}{OLD} = -e \Rightarrow \frac{p}{-12} = \frac{-2}{3} \Rightarrow p = 8$.

New initial speed = $16\vec{i} + 8\vec{j}$
Find length of hop means find Sx when Sy = 0.

$S_y = 0 \Rightarrow 8t - \frac{1}{2}(\frac{3}{5}g)t^2 = 0 \Rightarrow t = 0$ or $t = \frac{80}{3g}$.

At $t = \frac{80}{3g}$, $S_x = 16t + \frac{1}{2}(\frac{4}{5}g)t^2 = 16(\frac{80}{3g}) + \frac{1}{2}(\frac{4}{5}g)(\frac{6400}{9g^2}) = \frac{6400}{9g}$ metres.

9. contd ...   NO GAP HERE

At $t = \frac{80}{3g}$, $V_x = 16 + \frac{4}{5}gt = 16 + \frac{4}{5}g(\frac{80}{3g}) = \frac{112}{3}$

At $t = \frac{80}{3g}$, $V_y = 8 - \frac{3}{5}gt = 8 - \frac{3}{5}g(\frac{80}{3g}) = -8$.

If L is the landing angle, $\tan L = \frac{-V_y}{V_x} = \frac{8}{112/3} = \frac{3}{14}$.

$\Rightarrow \tan L = 0.2143 \Rightarrow L = 12^\circ\ 6'$.

10.

Before	After
$u\sin A\vec{i} - u\cos A\vec{j}$	$u\sin A\vec{i} + p\vec{j}$

$\frac{NEW}{OLD} = -e \Rightarrow \frac{p}{-u\cos A} = \frac{-1}{4} \Rightarrow p = \frac{1}{4}u\cos A$

Initial Speed = $u\sin A\vec{i} + \frac{1}{4}u\cos A\vec{j}$.

$S_y = 0 \Rightarrow \frac{1}{4}u\cos At - \frac{1}{2}g\cos At^2 = 0 \Rightarrow t = 0$ or $t = \frac{u}{2g}$

At $t = \frac{u}{2g}$, $S_x = u\sin At + \frac{1}{2}g\sin A\,t^2 = u\sin A(\frac{u}{2g}) + \frac{1}{2}g\sin A(\frac{u^2}{4g^2})$

$= \dfrac{5u^2 \sin A}{8g}$.

At the first hop (when $t = \frac{u}{2g}$),

$V_x = u\sin A + g\sin At = u\sin A + g\sin A(\frac{u}{2g}) = \frac{3}{2}u\sin A$.

$V_y = \frac{1}{4}u\cos A - g\cos At = \frac{1}{4}u\cos A - g\cos A(\frac{u}{2g}) = -\frac{1}{4}u\cos A$

After the first hop, $V_x = \frac{3}{2}u\sin A$ and $V_y = -\frac{1}{4}(-\frac{1}{4}u\cos A) = \frac{1}{16}u\cos A$

Second hop : Initial speed = $\frac{3}{2}u\sin A\vec{i} + \frac{1}{16}u\cos A\vec{j}$.

$S_y = 0 \Rightarrow \frac{1}{16}u\cos At - \frac{1}{2}g\cos At^2 = 0$

$\Rightarrow t = 0$ or $t = \frac{u}{8g}$.

At $t = \frac{u}{8g}$, $S_x = \frac{3}{2}u\sin A\,t + \frac{1}{2}g\sin A\,t^2$

$= \frac{3}{2}u\sin A(\frac{u}{8g}) + \frac{1}{2}g\sin A(\frac{u^2}{64g^2}) = \dfrac{25u^2\sin A}{128g}$

EXERCISE 7C

1. (i)

Before	(Mass)	After
$3\vec{i}$	2	$p\vec{i}$
$2\vec{i}$	1	$q\vec{i}$

$2(3) + 1(2) = 2p + q \Rightarrow 2p + q = 8$.

$\frac{p - q}{3 - 2} = -\frac{1}{2} \Rightarrow 2p - 2q = -1$.

Solving these gives $p = 2\frac{1}{2}$, $q = 3$,
Velocities are $2\frac{1}{2}\vec{i}$, $3\vec{i}$

(ii) K.E.$_{Before} = \frac{1}{2}(2)(9) + \frac{1}{2}(1)(4) = 11$J

K.E.$_{After} = \frac{1}{2}(2)(\frac{25}{4}) + \frac{1}{2}(1)(9) = 10\frac{3}{4}$J.

Loss = $11 - 10\frac{3}{4} = \frac{1}{4}$J.

(i) Before	(Mass)	After
$2\vec{i}$	M	$p\vec{i}$
$-6\vec{i}$	M	$q\vec{i}$

$1(2) + M(-6) = Mp + Mq \Rightarrow p + q = -4$,

$\frac{p-q}{2+6} = -\frac{3}{4} \Rightarrow p - q = -6$.

Solving these gives $p = -5$, $q = 1$

(ii) $\vec{I_1} = M\vec{v_1} - M\vec{u_1} = M(-5\vec{i}) - M(2\vec{i}) = -7M\vec{i}$

$\vec{I_2} = M\vec{v_2} - M\vec{u_2} = M(1\vec{i}) - M(-6\vec{i}) = 7M\vec{i}$.

ii) $K.E._{Before} = \frac{1}{2}(M)(2)^2 + \frac{1}{2}(M)(-6)^2 = 20M$ J.

$K.E._{After} = \frac{1}{2}(M)(-5)^2 + \frac{1}{2}(M)(1)^2 = 13M$ J

% Loss $= \frac{7M}{20M} \times \frac{100}{1} = 35\%$

(i) Before	(Mass)	After
$10\vec{i}$	3	$0\vec{i}$
$\vec{i}$	5	$q\vec{i}$

$3(10) + 5(1) = 3(0) + 5(q) \Rightarrow q = 7$,

$\frac{0-q}{10-1} = -e \Rightarrow e = \frac{q}{9} = \frac{7}{9}$.

(ii) 7 m/s.

iii) $\vec{I_1} = M\vec{v_1} - M\vec{u_1} = 3(0\vec{i}) - 3(10\vec{i}) = -30\vec{i}$ Ns.

$\vec{I_2} = M\vec{v_2} - M\vec{u_2} = 5(7\vec{i}) - 5(\vec{i}) = 30\vec{i}$ Ns.

$u = 7$, $v = 0$, $s = 2$, $a = ?$

$v^2 = u^2 + 2as \Rightarrow 0 = 49 + 2a(2) \Rightarrow a = \frac{-49}{4}$ m/s^2

(i) Before	(Mass)	After
$6\vec{i}$	5	$p\vec{i}$
$-4\vec{i}$	3	$q\vec{i}$

$5(6) + 3(-4) = 5p + 3q \Rightarrow 5p + 3q = 18$.

$\frac{p-q}{6+4} = -\frac{1}{3} \Rightarrow 3p - 3q = -10$.

Solving these gives $p = 1$, $q = \frac{13}{3}$.

(ii) Loss $= \frac{1}{2}Mu^2 - \frac{1}{2}Mv^2 = \frac{1}{2}(5)(6)^2 - \frac{1}{2}(5)(1)^2 = 87\frac{1}{2}$J.

ii) $M\vec{v} - M\vec{u} = 3(\frac{13}{3}\vec{i}) - 3(-4\vec{i}) = 25\vec{i}$ Ns.

(i) Before	(Mass)	After
$10\vec{i}$	2	$p\vec{i}$
$0\vec{i}$	3	$q\vec{i}$

$2(10) + 3(0) = 2p + 3q \Rightarrow 2p + 3q = 20$

$\frac{p-q}{10-0} = -\frac{1}{2} \Rightarrow p - q = -5$.

Solving these gives $p = 1$, $q = 6$.

Ans: 1 m/s and 6 m/s .

5. contd ...

(ii) $K.E._{Before} = \frac{1}{2}(2)(10)^2 + \frac{1}{2}(3)(0)^2 = 100$ J

$K.E._{After} = \frac{1}{2}(2)(1)^2 + \frac{1}{2}(3)(6)^2 = 55$ J

Loss $= 100 - 55 = 45$ J

(iii) To find the acceleration : $v = u + at \Rightarrow 0 = 6 + a(2) \Rightarrow a = -3$.

To find the force : $F = Ma \Rightarrow F = (3)(-3) = -9$ Newtons.

(i.e. it is a resistance force of 9N)

To find the distance : $v^2 = u^2 + 2as \Rightarrow 0 = (6)^2 + 2(-3)s \Rightarrow s = 6$ m.

To find work done : $W = Fs = 9(6) = 54$ J

To find power : Power $= \frac{WORK}{TIME} = \frac{54}{2} = 27$ Watts.

6. (a) $v^2 = u^2 + 2as \Rightarrow v^2 = 0^2 + 2(9.8)(19.6) \Rightarrow v = 19.6$

Rebound speed $= (0.8)(19.6) = 15.68$

$v^2 = u^2 + 2as \Rightarrow 0^2 = (15.68)^2 + 2(-9.8)s \Rightarrow s = 12.544$m.

(b) Gun Bullet Gun Bullet

$M_1u_1 + M_2u_2 = M_1v_1 + M_2v_2$

$\Rightarrow (2)(0) + (0.01)0 = (2)v_1 + (0.01)(300)$

$\Rightarrow v_1 = -1.5$ m/s $\Rightarrow$ initial speed of the gun $= 1.5$ m/s.

To find acceleration : $v^2 = u^2 + 2as = (0)^2 = (-1.5)^2 + 2a(0.05)$

$\Rightarrow a = -22.5$ m/s.

$F = ma \Rightarrow F = (2)(-22.5) = -45$ N.

A constant force of 45 N is required.

7. (i)

	Before	(Mass)	(After)
A:	$4\vec{i}$	M	$p\vec{i}$
B:	$0\vec{i}$	M	$q\vec{i}$

$M(4) + M(0) = Mp + Mq \Rightarrow p + q = 4$.

$\frac{p-q}{4-0} = -\frac{1}{2} \Rightarrow p - q = -2$.

Solving these gives $p = 1$, $q = 3$. Speed of B is 3 m/s.

(ii)

	Before	(Mass)	(After)
B:	$3\vec{i}$	M	$a\vec{i}$
C:	$0\vec{i}$	M	$b\vec{i}$

$M(3) + M(0) = Ma + Mb \Rightarrow a + b = 3$,

$\frac{a-b}{3-0} = -\frac{1}{2} \Rightarrow a - b = -\frac{3}{2}$.

Solving these gives $a = \frac{3}{4}$, $b = 2\frac{1}{4}$, Speed of B is $\frac{3}{4}$ m/s.

Yes, because A will catch up with B, since $V_A > V_B$

8. (i)

Before	(Mass)	After
$2\vec{i}$	M	$11k\vec{i}$
$\vec{i}$	M	$13k\vec{i}$

$M(2) + M(1) = M(11k) + M(13k) \Rightarrow k = \frac{1}{8}$.

$\therefore$ Their speeds will be $\frac{11}{8}$ and $\frac{13}{8}$.

(ii) $\dfrac{\frac{11}{8} - \frac{13}{8}}{2 - 1} = -e \Rightarrow e = \frac{1}{4}$

9.

Before	(Mass)	After
$U\vec{i}$	M	$0\vec{i}$
$-V\vec{i}$	3M	$q\vec{i}$

$M(U) + 3M(-V) = M(0) + 3M(q) \Rightarrow q = \dfrac{U - 3V}{3}$

$\dfrac{0 - q}{U + V} = -e \Rightarrow e = \dfrac{q}{U+V} = \dfrac{U - 3V}{3U + 3V}$ q.e.d.

0. (i)

	Before	(Mass)	After
A:	$6\vec{i}$	M	$p\vec{i}$
B:	$0\vec{i}$	M	$q\vec{i}$

$M(6) + M(0) = Mp + Mq \Rightarrow p + q = 6$,

$\dfrac{p - q}{6 - 0} = \dfrac{-2}{3} \Rightarrow p - q = -4$.

Solving these gives p = 1, q = 5

Their speeds are (1, 5, 0).

(ii)

	Before	(Mass)	After
B:	$5\vec{i}$	M	$a\vec{i}$
C:	$0\vec{i}$	M	$b\vec{i}$

$M(5) + M(0) = Ma + Mb \Rightarrow a + b = 5$.

$\dfrac{a - b}{5 - 0} = \dfrac{-2}{3} \Rightarrow a - b = \dfrac{-10}{3}$.

Solving these gives $a = \frac{5}{6}$, $b = \frac{25}{6}$.

Their speeds now are (1, $\frac{5}{6}$, $\frac{25}{6}$).

(iii)

	Before	(Mass)	After
A:	$1\vec{i}$	M	$c\vec{i}$
B:	$5/6\vec{i}$	M	$d\vec{i}$

$M(1) + M(5/6) = Mc + Md \Rightarrow c + d = \frac{11}{6}$,

$\dfrac{c - d}{1 - 5/6} = -\frac{2}{3} \Rightarrow c - d = -\frac{1}{9}$.

10. contd ...

Solving these gives: $c = \frac{31}{36}$, $d = \frac{35}{36}$

Their speeds are ($\frac{31}{36}$, $\frac{35}{36}$, $\frac{25}{6}$),

Since $V_A < V_B < V_C$, there will be no further collisions.

11.

Before	(Mass)	After
$5\vec{i}$	1	$p\vec{i}$
$\vec{i}$	2	$q\vec{i}$

$1(5) + 2(1) = 1(p) + 2(q) \Rightarrow p + 2q = 7 \Rightarrow p = 7-2q$.

$\dfrac{p - q}{5 - 1} = -e \Rightarrow -4e = p - q = (7 - 2q) - q = 7 - 3q$.

$\therefore e = \dfrac{3q - 7}{4}$

If there are to be no more collisions $V_B \leq V_C$

$\therefore q \leq 3$

If $q \leq 3$ then $e = \dfrac{3q - 7}{4} \leq \dfrac{3(3) - 7}{4} = \frac{1}{2}$

Ans: MAX VAlue = $\frac{1}{2}$

12. (a)

Before	(Mass)	After
$10\vec{i}$	10	$p\vec{i}$
$-5\vec{i}$	50	$q\vec{i}$

$10(10) + 50(-5) = 10p + 50q \Rightarrow p + 5q = -15$.

$\dfrac{p - q}{10 + 5} = -\frac{1}{2} \Rightarrow 2p - 2q = -15$.

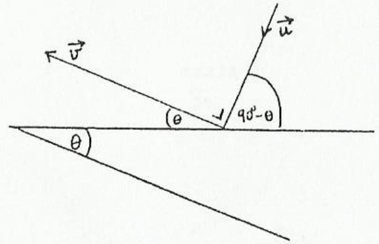
Solving these gives $p = -\frac{35}{4}$, $q = -\frac{5}{4}$.

The speeds are $8\frac{3}{4}$ m/s and $1\frac{1}{4}$ m/s.

$\vec{I_1} = M\vec{v} - M\vec{u} = 10\ (-\frac{35}{4}) - 10(10) = -187.5$ Ns.

The magnitude of the impulse = 187.5 Ns.

(b)



. (b) contd ...

$\vec{u} = -u \cos(90^\circ - \theta)\vec{i} - u \sin(90^\circ - \theta)\vec{j} = -u \sin\theta\vec{i} - u \cos\theta\vec{j}$.

$\vec{v} = -v \cos\theta\,\vec{i} + v \sin\theta\vec{j}$.

Before	(Mass)	After
, $\sin\theta\vec{i} - u\cos\theta\vec{j}$	M	$-v\cos\theta\,\vec{i} + v\sin\theta\,\vec{j}$.

$\vec{i}$ - velocity is unchanged $\Rightarrow -u\sin\theta = -v\cos\theta$
$\Rightarrow u\sin\theta = v\cos\theta$

$\frac{NEW}{OLD} = -e \Rightarrow \frac{v\sin\theta}{-u\cos\theta} = \frac{-2}{3} \Rightarrow 3v\sin\theta = 2u\cos\theta$

Dividing 2 by 1 gives $\frac{3v}{u} = \frac{2u}{v} \Rightarrow v^2 = \frac{2}{3}u^2$

$\Rightarrow \frac{1}{2}Mv^2 = \frac{2}{3}(\frac{1}{2}Mu^2) \Rightarrow \frac{2}{3}$ of the energy is preserved.

$\Rightarrow \frac{1}{3}$ of the energy has been lost.
(some Lucozade required).

ERCISE 7.D
==========

Before	(Mass)	After
$4\vec{i} + 3\vec{j}$	M	$p\vec{i} + 3\vec{j}$
$\vec{i} + 2\vec{j}$	M	$q\vec{i} + 2\vec{j}$

(4) + M (1) = M (p) + M (q) $\Rightarrow p + q = 5$

$\frac{p - q}{4 - 1} = -\frac{1}{3} \Rightarrow p - q = -1$

olving these gives p = 2, q = 3.

ne new velocities are $2\vec{i} + 3\vec{j}$ and $3\vec{i} + 2\vec{j}$

.E.$_{before}$ = $\frac{1}{2}(M)(4^2 + 3^2) + \frac{1}{2}(M)(1^2 + 2^2)$ = 15M J.

.E.$_{after}$ = $\frac{1}{2}M(2^2 + 3^2) + \frac{1}{2}M(3^2 + 2^2)$ = 13M J.

oss = 15M - 13M = 2M J.

Before	(Mass)	After
$3\vec{i} + 4\vec{j}$	2	$p\vec{i} + 4\vec{j}$
$-4\vec{i} + 3\vec{j}$	3	$q\vec{i} + 3\vec{j}$

2(3) + 3(-4) = 2(p) + 3(q) $\Rightarrow$ 2p + 3q = -6.

$\frac{p - q}{3 + 4} = \frac{-3}{7} \Rightarrow p - q = -3$

olving these gives p = -3, q = 0

i) Their velocities are $-3\vec{i} + 4\vec{j}$, $0\vec{i} + 3\vec{j}$.

i) K.E.$_{before}$ = $\frac{1}{2}(2)(3^2 + 4^2) + \frac{1}{2}(3)((-4)^2 + 3^2)$ = $62\frac{1}{2}$ J.

K.E.$_{after}$ = $\frac{1}{2}(2)((-3)^2 + 4^2) + \frac{1}{2}(3)(0^2 + 3^2)$ = $38\frac{1}{2}$ J.

Loss = $62\frac{1}{2} - 38\frac{1}{2}$ = 24J

2. contd ...

(iii) $\vec{I_1} = M\vec{v} - M\vec{u} = 2(-3\vec{i} + 4\vec{j}) - 2(3\vec{i} + 4\vec{j}) = -12\vec{i}$ Ns.

The magnitude of the impulse is 12 Ns.

3.

Before	(Mass)	After
$6\vec{i} + \vec{j}$	M	$0\vec{i} + \vec{j}$
$-2\vec{i} - 5\vec{j}$	2M	$q\vec{i} - 5\vec{j}$

M(6) + 2M(-2) = M(0) + 2Mq $\Rightarrow$ q = 1.

(i) It's velocity is $\vec{i} - 5\vec{j}$.

(ii) $\frac{0 - 1}{6 + 2} = -e \Rightarrow e = \frac{1}{8}$

4.

Before	(Mass)	After
$5\vec{i} + 5\vec{j}$	2M	$p\vec{i} + 5\vec{j}$
$0\vec{i} + 0\vec{j}$	M	$q\vec{i} + 0\vec{j}$

2M (5) + M (0) = 2Mp + Mq $\Rightarrow$ 2p + q = 10.

$\frac{p - q}{5 - 0} = -\frac{1}{2} \Rightarrow 2p - 2q = -5$.

Solving these gives $p = 2\frac{1}{2}$, q = 5.

(i) Their velocities are $2\frac{1}{2}\vec{i} + 5\vec{j}$; $5\vec{i} + 0\vec{j}$.

(ii) $\vec{I_1} = M\vec{v} - M\vec{u} = 2M(2\frac{1}{2}\vec{i} + 5\vec{j}) - 2M(5\vec{i} + 5\vec{j}) = -5m\vec{i}$ Ns.

$\vec{I_2} = M(5\vec{i} + 0\vec{j}) - M(0\vec{i} + 0\vec{j}) = 5M\vec{i}$ Ns.

(iii) K.E.$_{before}$ = $\frac{1}{2}(2M)(5^2 + 5^2) + \frac{1}{2}(M)(0^2 + 0^2)$ = 50M J.

K.E.$_{after}$ = $\frac{1}{2}(2M)((\frac{5}{2})^2 + 5^2) + \frac{1}{2}(M)(5^2 + 0^2)$ = $43\frac{3}{4}$M J.

Percentage Loss = $\frac{6\frac{1}{4}M}{50M} \times \frac{100}{1} = 12\frac{1}{2}\%$

(iv) $m_1 = \frac{5}{5} = 1$, $m_2 = \frac{5}{2\frac{1}{2}} = 2$.

$\therefore \tan\theta = \pm\frac{1 - 2}{1 + (1)(2)} = \pm\frac{1}{3} = 0.3333$

$\therefore \theta = 18^\circ 26'$ (Since θ is acute)

5. (i) $\vec{u_1} = 5\cos\theta\vec{i} + 5\sin\theta\vec{j} = 5(\frac{4}{5})\vec{i} + 5(\frac{3}{5})\vec{j} = 4\vec{i} + 3\vec{j}$

$\vec{u_2} = -4\sqrt{2}\cos 45^\circ\vec{i} + 4\sqrt{2}\sin 45^\circ\vec{j} = -4\vec{i} + 4\vec{j}$

Before	(Mass)	After
$4\vec{i} + 3\vec{j}$	2	$p\vec{i} + 3\vec{j}$
$-4\vec{i} + 4\vec{j}$	3	$q\vec{i} + 4\vec{j}$

2(4) + 3(-4) = 2p + 3q $\Rightarrow$ 2p + 3q = -4

$\frac{p - q}{4 + 4} = -\frac{7}{8} \Rightarrow p - q = -7$. Solving these gives p = -5, q = 2.

. (ii)

$-5\vec{i} + 3\vec{j}, \quad 2\vec{i} + 4\vec{j}$

(iii) $K.E._{\text{before}} = \frac{1}{2}(2)\ (4^2 + 3^2) + \frac{1}{2}(3)\ (\ (-4)^2 + 4^2) = 73$ J

$K.E._{\text{after}} = \frac{1}{2}(2)\ ((-5)^2 + 3^2) + \frac{1}{2}(3)\ (2^2 + 4^2) = 64$ J

Loss $= 73 - 64 = 9$J

.

Before	(Mass)	(After)
$5\vec{i} + 4\vec{j}$	5	$p\vec{i} + 4\vec{j}$
$-2\vec{i} - 3\vec{j}$	10	$q\vec{i} - 3\vec{j}$

$5(5) + 10(-2) = 5p + 10q \Rightarrow p + 2q = 1$.

$\frac{p - q}{5 + 2} = -\frac{1}{7} \Rightarrow p - q = -1$.

Solving these gives: $p = -\frac{1}{3}, \quad q = \frac{2}{3}$.

(i) $-\frac{1}{3}\vec{i} + 4\vec{j}$ (ii) $\frac{2}{3}\vec{i} - 3\vec{j}$

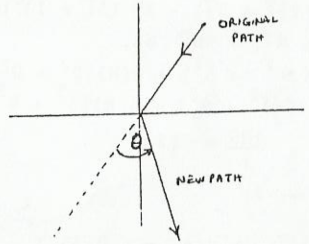
$K.E._{\text{before}} = \frac{1}{2}(5)\ (25 + 16) + \frac{1}{2}(10)\ (4 + q) = 102.5 + 65 = 167.5$ J

$K.E._{\text{after}} = \frac{1}{2}(5)\ (\frac{1}{9} + 16) + \frac{1}{2}(10)\ (\frac{4}{9} + 9) = \frac{725}{18} = \frac{850}{18} = \frac{1575}{18}$

$= 87.5$ J.

Loss $= 167.5 - 87.5 = 80$J. Q.E.D.

$M_1 = \frac{-3}{-2} = \frac{3}{2}$; $M_2 = \frac{-3}{\frac{2}{3}} = \frac{-9}{2}$



Tan $\theta = \pm \frac{\frac{3}{2} + \frac{9}{2}}{1 - \frac{27}{4}} = \pm \frac{24}{23}$

Tan $\theta = \frac{24}{23}$, since θ is acute.

.

Before	(Mass)	After
$u\text{ Cos } A\vec{i} + u\text{Sin}A\vec{j}$	M	v Cos Bi + v Sin B$\vec{j}$
$0\vec{i} + 0\vec{j}$	M	$q\vec{i} + 0\vec{j}$

1. USinA = VSinB

2. $M(u\text{CosA}) + M(0) = M(v\text{CosB}) + Mq \Rightarrow q = u\text{CosA} - v\text{Cos B}$

3. $\frac{v\text{CosB} - q}{u\text{CosA} - 0} = -\frac{1}{4} \Rightarrow 4v\text{CosB} - 4q = -u\text{CosA}$

But $q = u\text{ CosA} - v\text{CosB}$

$\therefore$ 3 reads : $4v\text{CosB} - 4u\text{CosA} + 4v\text{CosB} = -u\text{CosA}$

$\Rightarrow 3u\text{CosA} = 8v\text{CosB}$ ...... equation 4.

Dividing equation by equation, we get:

$\frac{u\text{SinA}}{3u\text{ CosA}} = \frac{v\text{SinB}}{8v\text{CosB}} \Rightarrow 8\text{ Tan A} = 3\text{ TanB}$ Q.E.D.

8. (a)

Before	(Mass)	After
$U_1\vec{i}$	3	$V_1\vec{i}$
$U_2\vec{i}$	4	$q\vec{i}$

1: $3U_1 + 4U_2 = 3V_1 + 4q \Rightarrow q = \frac{1}{4}(3U_1 + 4U_2 - 3V_1)$

2: $\frac{V_1 - q}{U_1 - U_2} = -e \Rightarrow V_1 - q = -eU_1 + eU_2$

But $q = \frac{1}{4}(3U_1 + 4U_2 - 3V_1)$

$\therefore V_1 - \frac{1}{4}(3U_1 + 4U_2 - 3V_1) = -eU_1 + eU_2$

$\Rightarrow 4V_1 - 3U_1 - 4U_2 + 3V_1 = -4eU_1 + 4eU_2$

$\Rightarrow 7V_1 = U_1\ (3 - 4e) + 4U_2\ (1 + e)$ Q.E.D.

$I = 3V_1 - 3U_1$. But $V_1 = \frac{1}{7}(U_1(3 - 4e) + 4U_2(1 + e))$

$\therefore I = \frac{3}{7}(U_1(3 - 4e) + 4U_2(1+e)) - 3U_1$

$= \frac{3}{7}\ (3U_1 - 4eU_1 + 4U_2 + 4eU_2 - 7U_1)$

$= \frac{3}{7}\ (-4U_1 - 4eU_1 + 4U_2 + 4eU_2)$

$= \frac{12}{7}\ (-U_1 - eU_1 + U_2 + eU_2)$

$= \frac{12}{7}\ (-U_1\ (1 + e) + U_2\ (1 + e)\ )$

$= \frac{12}{7}\ (1 + e)\ (U_2 - U_1)$ Q.E.D.

(b) The sphere of mass M will travel along the $\vec{i}$-axis, therefore the sphere of mass 4kg will move along the $\vec{j}$-axis.

Before	(Mass)	After
$U\text{Cos}A\vec{i} + U\text{Sin}A\vec{j}$	4	$0\vec{i} + U\text{Sin}A\vec{j}$
$0\vec{i} + 0\vec{j}$	M	$q\vec{i} + 0\vec{j}$

$4(U\text{CosA}) + M(0) = 4(0) + Mq \Rightarrow Mq = 4U\text{CosA}$,

$\frac{0 - q}{U\text{CosA} - 0} = -\frac{4}{7} \Rightarrow 7q = 4U\text{CosA}$.

It follows that $7q = Mq \Rightarrow M = 7$.

9.

Before	(Mass)	After
$8\vec{i} + 4\vec{j}$	M	$p\vec{i} + 4\vec{j}$
$0i + 0j$	2M	$q\vec{i} + 0\vec{j}$

Since $8\vec{i} + 4\vec{j}$ is perpendicular to $p\vec{i} + 4\vec{j}$

$\frac{4}{8} \times \frac{4}{p} = -1 \Rightarrow -8p = 16 \Rightarrow p = -2$

(i) Its velocity after impact is $-2\vec{i} + 4\vec{j}$.

(ii) 1: $M(8) + 2(0) = M(-2) + 2M(q) \Rightarrow q = 5$.

It's velocity is $5\vec{i} + 0\vec{j}$.

/...

contd ...

ii) $\frac{p - q}{8 - 0} = -e \Rightarrow \frac{-2 - 5}{8 - 0} = -e \Rightarrow e = 7/8$

Before	(Mass)	After
$2UCosA\vec{i} + 2USinA\vec{j}$	$2M$	$p\vec{i} + 2USinA\vec{j}$
$-U\vec{i} + 0\vec{j}$	M	$0\vec{i} + 0\vec{j}$

$2M(2UCosA) + M(-U) = 2M(p) + M(0) \Rightarrow p = 2UCosA - \frac{1}{2}U.$

$\frac{p - 0}{2UCosA + U} = \frac{-5}{118.} \Rightarrow 118p = -10UCosA - 5U$

But $p = 2UCosA - \frac{1}{2}U \qquad \therefore\ 118(2UCosA - \frac{1}{2}U) = -10UCosA - 5U$

$\Rightarrow 236CosA - 59 = -10CosA - 5 \Rightarrow CosA = \frac{9}{41} \Rightarrow SinA = \frac{40}{41}$

$p = 2UCosA - \frac{1}{2}U = 2U(\frac{9}{41}) - \frac{1}{2}U = \frac{-5U}{82}$

$2USinA = 2U(\frac{40}{41}) = \frac{80U}{41} = \frac{160U}{82}$

$\therefore$ Velocity after impact $= \frac{U}{82}(-5\vec{i} + 160\vec{j})$

(i)

Before	(Mass)	After
$2u\vec{i}$	M	$p\vec{i}$
$u\vec{i}$	M	$q\vec{i}$

$M(2u) + M(u) = M(p) + M(q) \Rightarrow p + q = 3u$

$\frac{p - q}{2u - u} = -e \Rightarrow p - q = -eu$

lving these gives $p = \frac{u(3 - e)}{2}, \quad q = \frac{u(3 + e)}{2}$

$K.E._{Before} = \frac{1}{2}M(2u)^2 + \frac{1}{2}M(u)^2 = \frac{5}{2}Mu^2$

$K.E._{After} = \frac{1}{2}M(\frac{u(3-e)}{2})^2 + \frac{1}{2}M(\frac{u(3 + e)}{2})^2$

$= \frac{1}{8}Mu^2(9 - 6e + e^2 + 9 + 6e + e^2) = \frac{1}{4}Mu^2(9 + e^2)$

e Loss $= \frac{5}{2}Mu^2 - \frac{1}{4}Mu^2(9 + e^2) = \frac{1}{4}Mu^2(10 - 9 - e^2)$

$= \frac{Mu^2(1-e^2)}{4}$

(ii)

Before	(Mass)	After
$\sqrt{3}\,u\vec{i} + u\vec{j}$	M	$p\vec{i} + u\vec{j}$
$0i + 0j$	M	$q\vec{i} + 0\vec{j}$

$(\sqrt{3}u) + M(0) = Mp + Mq \Rightarrow p + q = \sqrt{3}u$

$\frac{p - q}{\sqrt{3}u - 0} = -e \Rightarrow p - q = -\sqrt{3}eu$

/...

11. contd ...

Solving these gives $p = \frac{\sqrt{3}u(1 - e)}{2}, \qquad q = \frac{\sqrt{3}u(1 + e)}{2}$

$K.E._{Before} = \frac{1}{2}M(2u)^2 + \frac{1}{2}M(0)^2 = 2Mu^2$

$K.E._{After} = \frac{1}{2}M\left(\left(\frac{\sqrt{3}u(1 - e)}{2}\right)^2 + u^2\right) + \frac{1}{2}M\left(\frac{\sqrt{3}u(1 + e)}{2}\right)^2$

$= \frac{1}{2}M\left[\frac{3u^2(1-2e + e^2)}{4} + u^2 + \frac{3u^2(1 + 2e + e^2)}{4}\right]$

$= \frac{1}{8}Mu^2(3 - 6e + 3e^2 + 4 + 3 + 6e + 3e^2)$

$= \frac{1}{8}Mu^2(10 + 6e^2) = \frac{1}{4}Mu^2(5 + 3e^2)$

Loss $= 2Mu^2 - \frac{1}{4}Mu^2(5 + 3e^2) = Mu^2(2 - \frac{5}{4} - \frac{3}{4}e^2)$

$= \frac{Mu^2}{4}(3 - 3e^2) = \frac{3Mu^2(1 - e^2)}{4}$

= 3 times the loss of energy in (i)

CHAPTER 8. STATICS.

EXERCISE 8.A

1. (a) $R = 4\vec{j} + 5\vec{j} + \vec{j} = 10\vec{j}$ N

$4(0) + 5(50) + 1(100) = 10(x) \Rightarrow x = 35$ cm

Answer: 35 cm from p.

(b) $R = 2\vec{j} + \vec{j} + 2\vec{j} = 5\vec{j}$

$2(0) + 1(2) + 2(5) = 5x \Rightarrow 2.4m$

Answer: 2.4 m from p.

(c) $R = \vec{j} - 7\vec{j} + \vec{j} = -5j$

$1(0) - 7(1) + 1(2) = -5(x) \Rightarrow x = 1$

Answer: 1m from p.

(d) $R = 3\vec{j} - 9\vec{j} + 3\vec{j} = -3\vec{j}$

$3(1) - 9(3) + 3(5) = -3x \Rightarrow x = 3$

Answer: 3m from p.

(e) $R = \vec{j} - 2\vec{j} - 3\vec{j} + \vec{j} = -3j$

$1(0) - 2(2) - 3(5) + 1(6) = -3(x) \Rightarrow x = 4\frac{1}{3}$

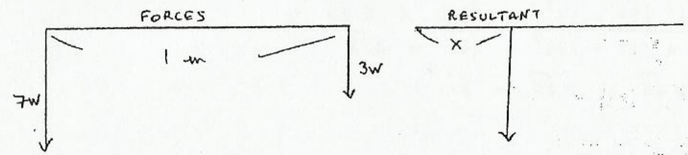
Answer: $4\frac{1}{3}$m from p.

2. $R = 2W + W + 3W = 6W$

$2W(0) + W(3) + 3W(6) = 6W(x) \Rightarrow x = 3\frac{1}{2}$.

Answer: $3\frac{1}{2}$M and $2\frac{1}{2}$M from the ends.

3.



/...

.. contd ...

$R = 7W + 3W = 10W$
$7W(0) + 3W(1) = 10W(x) \Rightarrow x = \frac{3}{10}m = 30cm$ = Answer.

.. Let the length of the plank be 1. Let d = the distance from the resultant's line of action from the left hand end.

$R = w + x$

$\therefore \; w(0) + x(1) = (w + x)d \Rightarrow d = \frac{x}{w+x}$

The remainder is $1 - \frac{x}{w+x} = \frac{w}{w+x}$

The ratio of these parts $= \frac{x}{w+x} : \frac{w}{w+x} = x : w$

.. (i) $R = 4W + W + KW = (5 + K)W$

Taking moments about p.

$4W(0) + W(1) + KW(2) = (5+K)W\,(\frac{7}{8})$

$\Rightarrow 1 + 2K = \frac{7(5+K)}{8} \Rightarrow 8 + 16K = 35 + 7K \Rightarrow K = 3.$

(ii) $4W(0) + W(1) + KW(2) = (5+K)W\,(\frac{11}{10})$

$\Rightarrow 1 + 2K = \frac{11(5+K)}{10} \Rightarrow 10 + 20K = 55 + 11K \Rightarrow K = 5.$

XERCISE 8.B

. $\left.\begin{array}{l} 3\text{ N at }(2, 1) \\ 2\text{ N at }(4, 3) \\ 1\text{ N at }(10, 9) \end{array}\right\} = 6\text{ N at }(x, y)$

$3(2) + 2(4) + 1(10) = 6(x) \Rightarrow x = 4$
$3(1) + 2(3) + 1(9) = 6(y) \Rightarrow y = 3$

Answer: (4, 3)

. $\left.\begin{array}{l} 1\text{ N at }(1, 1) \\ 2\text{ N at }(1, 7) \\ 3\text{ N at }(3, 1) \\ 4\text{ N at }(2, 3) \end{array}\right\} = 10\text{ N at }(x, y)$

$1(1) + 2(1) + 3(3) + 4(2) = 10(x) \Rightarrow x = 2$
$1(1) + 2(7) + 3(1) + 4(3) = 10(y) \Rightarrow y = 3$

Answer: (2, 3)

. $3(4) + 2(q) = 5x \Rightarrow x = 6$
$3(1) + 2(-q) = 5y \Rightarrow y = 3$ Answer: (6, − 3)

$3(4) + 2(q) + 1(x) = 6(6) \Rightarrow x = 6$
$3(1) + 2(-q) + 1(y) = 6(-2) \Rightarrow y = 3.$ Answer: (6, −3)

. $W(3) + 2W(12) = 3W(x) \Rightarrow x = 9$
$W(1) + 2W(19) = 3W(y) \Rightarrow y = 13.$ Answer: (9, 13) = g

$|pg| = \sqrt{(9 - 3)^2 + (13 - 1)^2} = \sqrt{180} = 6\sqrt{5}$
$|gq| = \sqrt{(12 - 9)^2 + (19 - 13)^2} = \sqrt{45} = 3\sqrt{5}$

$|pg| : |gq| = 6\sqrt{5} : 3\sqrt{5} = 2 : 1$

5. $\left.\begin{array}{l} 2\text{ N at }(1, 2) \\ 3\text{ N at }(1, 7) \\ 5\text{ N at }(5, -1) \end{array}\right\} = 10\text{N at }(x, y)$

$2(1) + 3(1) + 5(5) = 10(x) \Rightarrow x = 3$
$2(2) + 3(7) + 5(-1) = 10(y) \Rightarrow y = 2.$ Answer: $3\vec{i} + 2\vec{j}$

$\left.\begin{array}{l} 2\text{ N at }(1, 2) \\ 3\text{ N at }(1, 7) \\ 5\text{ N at }(5, -1) \\ 2\text{ N at }(x, y) \end{array}\right\} = 12\text{N at }(4, 1)$

$2(1) + 3(1) + 5(5) + 2(x) = 12(4) \Rightarrow x = 9$
$2(2) + 3(7) + 5(-1) + 2(y) = 12(1) \Rightarrow y = -4$ Ans: (9 − 4)

6. $1(4) + 2(1) + 3(K) = 6(2\frac{1}{2}) = K = 3.$
$1(1) + 2h + 3(1) = 6(4) = h = 10$

7. Centroid of triangle pqr $= (\frac{2 + 5 + 3}{3}, \frac{1 + 3 - 1}{3}) = (\frac{10}{3}, 1)$

$\left.\begin{array}{l} M\text{ at }(2, 1) \\ M\text{ at }(5, 3) \\ M\text{ at }(3, -1) \end{array}\right\} = 3M\text{ at }(x, y)$

$M(2) + M(5) + M(3) = 3M(x) \Rightarrow x = \frac{10}{3}$
$M(1) + M(3) + M(-1) = 3M(y) \Rightarrow y = 1$

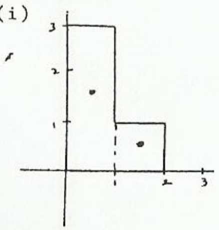
The centre of gravity is at $(\frac{10}{3}, 1)$ which is the centroid.

$M(2) + M(5) + M(3) + 2M(x) = 5M(2) \Rightarrow x = 0$
$M(1) + M(3) + M(-1) + 2M(-1) = 5M(1) \Rightarrow y = -1$

Answer: (0, − 1)

EXERCISE 8.C

(i)



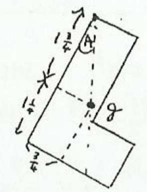
$\left.\begin{array}{l} 3\text{ at }(\frac{1}{2}, 1\frac{1}{2}) \\ 1\text{ at }(1\frac{1}{2}, \frac{1}{2}) \end{array}\right\} = 4\text{ at }(x, y)$

$3(\frac{1}{2}) + 1(1\frac{1}{2}) = 4x \Rightarrow x = \frac{3}{4}$
$3(1\frac{1}{2}) + 1(\frac{1}{2}) = 4y \Rightarrow y = 1\frac{1}{4}$

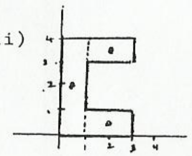
Answer: $(\frac{3}{4}, 1\frac{1}{4})$

$\text{Tan } A = \frac{\frac{3}{4}}{1\frac{3}{4}} = \frac{3}{7} = 0.4286$

$\therefore \; A = 23^o\,12'$



(ii)

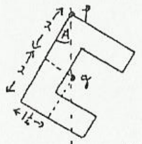


$\left.\begin{array}{l} 4\text{ at }(\frac{1}{2}, 2) \\ 2\text{ at }(2, \frac{1}{2}) \\ 2\text{ at }(2, 3\frac{1}{2}) \end{array}\right\} = 8\text{ at }(x, y)$

$4(\frac{1}{2}) + 2(2) + 2(2) = 8(x) \Rightarrow x = 1\frac{1}{4}$
$4(2) + 2(\frac{1}{2}) + 2(3\frac{1}{2}) = 8(y) \Rightarrow y = 2.$ Answer: $(1\frac{1}{4}, 2)$

/...

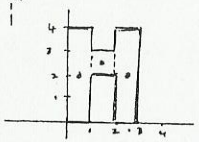
ii) contd ...



$\text{Tan } A = \frac{1\frac{1}{4}}{2} = \frac{5}{8} = 0.625$

$\therefore A = 32^\circ$

iii)

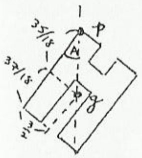


$$\left.\begin{array}{l} 4 \text{ at } (\tfrac{1}{2}, 2) \\ 1 \text{ at } (1\tfrac{1}{2}, 2\tfrac{1}{2}) \\ 4 \text{ at } (2\tfrac{1}{2}, 2) \end{array}\right\} = 9 \text{ at } (x, y)$$

$4(\frac{1}{2}) + 1(1\frac{1}{2}) + 4(2\frac{1}{2}) = 9(x) \Rightarrow x = \frac{3}{2}$

$4(2) + 1(2\frac{1}{2}) + 4(2) = 9(y) \Rightarrow y = \frac{37}{18}$

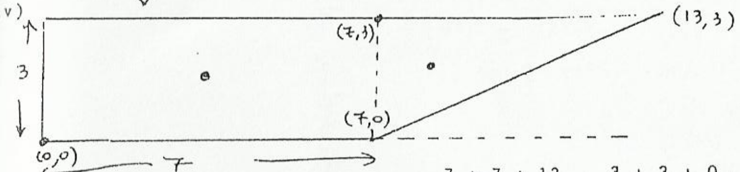
Answer: $(\frac{3}{2}, \frac{37}{18})$



$\text{Tan } A = \frac{3/2}{35/18} = \frac{27}{35} = 0.7714$

$\therefore A = 37^\circ\ 39'$

iv)



Triangle piece: Centroid is at $(\frac{7 + 7 + 13}{3}, \frac{3 + 3 + 0}{3}) = (9, 2)$

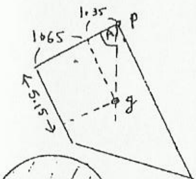
Area is $\frac{1}{2}(6)(3) = 9$ square units

$$\left.\begin{array}{l} 21 \text{ at } (3\tfrac{1}{2}, 1\tfrac{1}{2}) \\ 9 \text{ at } (9, 2) \end{array}\right\} = 30 \text{ at } (x, y)$$

$21(3\frac{1}{2}) + 9(9) = 30(x) \Rightarrow x = 5.15$

$21(1\frac{1}{2}) + 9(2) = 30(y) \Rightarrow y = 1.65$

Answer: $(5.15, 1.65)$



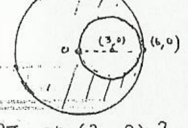
$\text{Tan } A = \frac{5.15}{1.35} = 3.8148$

$A = 75^\circ\ 19'$

Larger Circle: Area = 36π, Centre = $(0, 0)$

Smaller Circle: Area = 9π, Centre = $(3, 0)$

Remainder: Area = 27π, Centre of gravity is at (x, y)



$$\left.\begin{array}{l} 9\pi \text{ at } (3, 0) \\ 27\pi \text{ at } (x, y) \end{array}\right\} = 36\pi \text{ at } (0, 0)$$

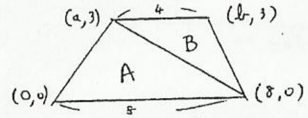
2. contd ...

$9\pi(3) + 27\pi(x) = 36(0) \Rightarrow x = -1$

$9\pi(0) + 27\pi(y) = 36(0) \Rightarrow y = 0$

Centre of gravity is at (-1, 0) which is 1 centimetre from the centre of the larger circle

3. We are interested in y-coordinates only.



Triangle A : Area = $\frac{1}{2}(8)(3) = 12$. Centre of gravity is at $(\frac{0 + 8 + a}{3}, \frac{0 + 0 + 3}{3}) = (k, 1)$

Triangle B : Area = $\frac{1}{2}(4)(3) = 6$

Centre of gravity is at $(\frac{a+b+3}{8}, \frac{3+3+0}{3}) = (h, 2)$

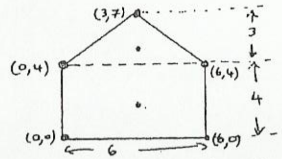
Whole piece : Area = 18 Centre of gravity is at (x, y)

$$\left.\begin{array}{l} 12 \text{ at } (k, 1) \\ 6 \text{ at } (h, 2) \end{array}\right\} = 18 \text{ at } (x, y)$$

Taking moments about the x-axis only.

$12(1) + 6(2) = 18y \Rightarrow y = \frac{4}{3}$cm. Answer: 4/3 cm.

4.



Rectangle A : Area = 6 x 4 = 24. Centre of gravity is at (3, 2)

Triangle B : area = $\frac{1}{2}(6)(3) = 9$. Centre of gravity is at $(\frac{0 + 3 + 6}{3}, \frac{4 + 4 + 7}{3}) = (3, 5)$

Lamina : Area = 24 + 9 = 33. Centre of gravity is at (x, y)

$$\left.\begin{array}{l} 24 \text{ at } (3, 2) \\ 9 \text{ at } (3, 5) \end{array}\right\} = 33 \text{ at } (x, y)$$

Taking moments about the x-axis

$24(2) + 9(5) = 33(y) \Rightarrow y = \frac{93}{33} = \frac{31}{11}$ cm

5. $|ab|^2 = |bd|^2 + |ad|^2 \Rightarrow 20^2 = 16^2 + |ad|^2 \Rightarrow |ad| = 12$

Triangle : Area = $\frac{1}{2}(32)(12) = 192$

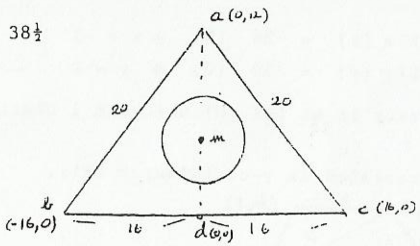
Taking d as the origin, the centre of gravity is at the centroid of a(0, 12) b(-16, 0) and c(16, 0) which is at $(\frac{0-16+16}{3}, \frac{12+0+0}{3}) = (0, 4)$

/...

. contd ...

Circle : Area = $\pi r^2 = \frac{22}{7} \times \frac{49}{4} = \frac{77}{2} = 38\frac{1}{2}$

Centre of gravity is at m(0, 6)



The remainder : Area = $192 - 38\frac{1}{2} = 153\frac{1}{2}$

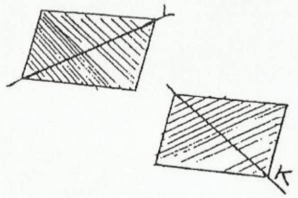
Centre of gravity is at (x, y)

$$\left.\begin{array}{l} 153\frac{1}{2} \text{ at } (x, y) \\ 38\frac{1}{2} \text{ at } (0, 6) \end{array}\right\} = 192 \text{ at } (0, 4)$$

$153\frac{1}{2}(x) + 38\frac{1}{2}(0) = 192\ (0) \Rightarrow x = 0$

$153\frac{1}{2}(y) + 38\frac{1}{2}(6) = 192(4) \Rightarrow y = \frac{1074}{307} = 3.5$ cm

.



Centre of gravity is along L, since L bisects each thin strip.

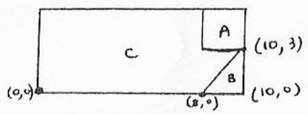
Centre of gravity is along K, for the same reason.

$\therefore$ Centre of gravity is where the diagonals meet.

. (a) $3(-2) + 4(1) + 5(3) + 6(4) = 18(x) \Rightarrow x = \frac{37}{18}$

$3(2) + 4(6) + 5(7) + 6(-4) = 18(y) \Rightarrow y = \frac{41}{18}$

(b)



A: Area is 3 x 2 = 6. Centre of gravity is at $(9, 4\frac{1}{2})$

B: Area = $\frac{1}{2}(2)(3) = 3$. Centre of gravity is at

$\left(\frac{8+10+10}{3}, \frac{0+0+3}{3}\right) = (28/3, 1)$

Rectangle: Area = 10 x 6 = 60 Centre of gravity is at (5, 3)

Remainder C: Area = 60 - 6 - 3 = 51. Centre of gravity is at (x, y)

$$\left.\begin{array}{l} 6 \text{ at } (9, 4\frac{1}{2}) \\ 3 \text{ at } (\frac{28}{3}, 1) \\ 51 \text{ at } (x, y) \end{array}\right\} = 60 \text{ at } (5, 3)$$

$6(9) + 3(\frac{28}{3}) + 51\ (x) = 60(5) \Rightarrow x = \frac{218}{51}$

$6(4\frac{1}{2}) + 3(1) + 51(y) = 60(3) \Rightarrow y = \frac{150}{51}$

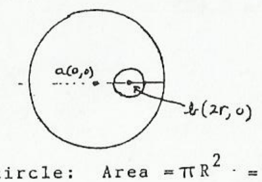
Ans: (218/51, 150/51)

8. (a)

$4(2) + 5(x) + 1(5) + 3(1) = 13(2) \Rightarrow x = 2$

$4(3) + 5(4) + 1(y) + 3(7) = 13(4) \Rightarrow y = -1$

(b)



Larger circle: Area = $\pi R^2 = \pi(4r)^2 = 16\pi r^2$

Centre of gravity is at (0, 0)

Smaller Circle : Area = πr^2

Centre of gravity is at b(2r, 0)

Remainder : Area = $16\pi r^2 - \pi r^2 = 15\pi r^2$
Centre of gravity is at (x, y)

$$\left.\begin{array}{l} 15\pi r^2 \text{ at } (x, y) \\ \pi r^2 \text{ at } (2r, 0) \end{array}\right\} = 16\pi r^2 \text{ at } (0, 0)$$

$15\pi r^2\ (x) + \pi r^2\ (2r) = 16\pi r^2\ (0) \Rightarrow x = -\frac{2r}{15}$.
$15\pi r^2\ (y) + \pi r^2\ (0) = 16\pi r^2\ (0) \Rightarrow y = 0.$

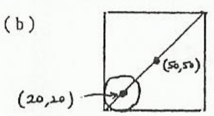
Answer: $(\frac{-2r}{15}, 0)$

9. (a) $5(3) + 8(4) + 3(-1) + 2(2) = 18(x) \Rightarrow x = \frac{8}{3}$

$5(-1) + 8(2) + 3(5) + 2(-6) = 18(y) \Rightarrow y = \frac{7}{9}$

Answer: $(\frac{8}{3}, \frac{7}{9})$

(b)



Circle: Area = $\pi r^2 = \frac{22}{7} \times \frac{400}{1} = 1257$

Centre of gravity is at (20, 20)

Square: Area = 100 x 100 = 10,000
Centre of gravity is at (50, 50)

Remainder: Area = 10,000 - 1257 = 8743
Centre of gravity is at (x, y)

$$\left.\begin{array}{l} 1257 \text{ at } (20, 20) \\ 8743 \text{ at } (x, y) \end{array}\right\} = 10{,}000 \text{ at } (50, 50)$$

$1257(20) + 8743(x) = 10{,}000(50) \Rightarrow x = 54.3$

Answer: 54.

'. Distance $= \frac{115}{2} = 57.5$ cm

Whole rectangle : Area $= 90 \times 115 = 10,350$

Centre of gravity is at (45, 75.5)

Larger Section : Area $= 80 \times 70 = 5,600$

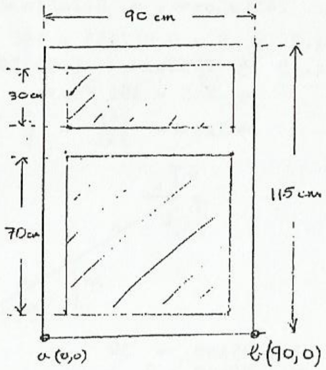
Centre of gravity is at (45, 40)

Smaller Section: Area $= 80 \times 30 = 2,400$

Centre of gravity is at (45, 95)

Remainder: area $= 10,350 - 5,600 - 2,400 = 2,350$

Centre of gravity is at (x, y)



$$\left.\begin{array}{l} 5,600 \text{ at } (45, 40) \\ 2,400 \text{ at } (45, 95) \\ 2,350 \text{ at } (x, y) \end{array}\right\} = 10,350 \text{ AT } (45, 57.5)$$

Taking moments about the x-axis:

$5,600(40) + 2,400(95) + 2,350(y) = 10,350\ (57.5)$

$\Rightarrow y = 60.9$ cm

ERCISE 8.D
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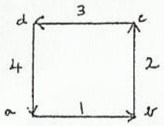


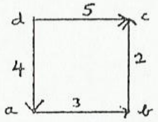
$\vec{R} = \vec{i} + 2\vec{j} - 3\vec{i} - 4\vec{j} = -2\vec{i} - 2\vec{j}$

$|\vec{R}| = \sqrt{(-2)^2 + (-2)^2} = \sqrt{8}$ N

$\vec{ab} = 3\sqrt{2}\left(\frac{1}{\sqrt{2}}\vec{i}\right) - 3\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)\vec{j} = 3\vec{i} - 3\vec{j}$

e new resultant $= (-2\vec{i} - 2\vec{j}) + (3\vec{i} - 3\vec{j}) = \vec{i} - 5\vec{j}$

$\therefore |\vec{R}| = \sqrt{1^2 + (-5)^2} = \sqrt{26}$ N



$\vec{R} = 3\vec{i} - 2\vec{j} + 5\vec{i} - 4\vec{j} = 8\vec{i} - 6\vec{j}$

$|\vec{R}| = \sqrt{8^2 + (-6)^2} = \sqrt{100} = 10$ N

et x = the distance of it's line of action from $\underline{a}$.

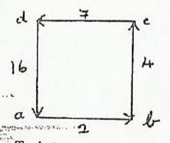
he moment of the sum = the sum of the moments. (Taking moments about $\underline{a}$)

$10(x) = 3(0) - 2(1) - 5(1) + 4(0) \Rightarrow x = \frac{7}{10}$ m $= 70$cm

et it intersect at a distance $\underline{k}$ from $\underline{a}$, therefore a distance $(1-k)$ from $\underline{b}$.

aking moments about the point of intersection.

$0(0) = 3(0) - 2(1-k) - 5(1) + 4(k) \Rightarrow k = \frac{7}{6}$m. Ans: $1\frac{1}{6}$m from $\underline{a}$.



(i) $\vec{R} = 2\vec{i} + 4\vec{j} - 7\vec{i} - 16\vec{j} = -5\vec{i} - 12\vec{j}$

(ii) $\therefore |\vec{R}| = \sqrt{(-5)^2 + (-12)^2} = \sqrt{169} = 13$ N

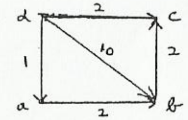
iii) Taking moments about $\underline{a}$:

/...

3. contd ...

$13(x) = 2(0) + 4(1) + 7(1) + 16(0) \Rightarrow x = \frac{11}{13}$ m.

4.



(i) $\vec{R} = 2\vec{i} + 2\vec{j} + 2\vec{i} - \vec{j} + (8\vec{i} - 6\vec{j}) = 12\vec{i} - 5\vec{j}$

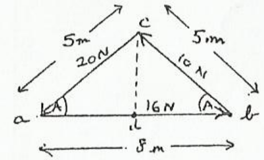
$\therefore |\vec{R}| = \sqrt{12^2 + (-5)^2} = \sqrt{169} = 13$ N

(ii) Taking moments about d:

$13(x) = 2(3) + 2(4) + 2(0) + 1(0) + 10(0) \Rightarrow x = \frac{14}{13}$ m.

(iii) $13x = 2(3) + 2(4) + 2(0) + 1(0) + 10(0) + 12 \Rightarrow x = 2$m.

5.



$|ad|^2 + |dc|^2 = |ac|^2$

$\Rightarrow 4^2 + |dc|^2 = 5^2$

$\Rightarrow |dc| = 3$m

$\therefore$ Tan A $= \frac{3}{4}$

$\therefore$ SinA $= \frac{3}{5}$, Cos A $= \frac{4}{5}$

$\vec{ab} = 16\vec{i}$

$\vec{bc} = -10\text{Cos}A\vec{i} + 10\text{Sin}A\vec{j} = -8\vec{i} + 6\vec{j}$

$\vec{ca} = -20\text{Cos}A\vec{i} - 20\text{Sin}A\vec{j} = -16\vec{i} - 12\vec{j}$

$\Rightarrow \vec{R} = -8\vec{i} - 6\vec{j}$

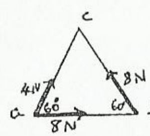
$\therefore |\vec{R}| = \sqrt{(-8)^2 + (-6)^2} = \sqrt{100} = 10$ N

Taking moments about c :

Moment of the sum = the sum of the moments

$10(x) = 16(3) + 10(0) + 20(0) \Rightarrow x = 4.8$m .

6.



(i) $\vec{ab} = 8\vec{i}$

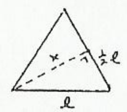
$\vec{bc} = -8\text{Cos}60^\circ\vec{i} + 8\text{Sin}60^\circ\vec{j} = -8(\frac{1}{2})\vec{i} + 8(\frac{\sqrt{3}}{2})\vec{j} = -4\vec{i} + 4\sqrt{3}\vec{j}$

ca $\vec{ac} = 4\text{Cos}60^\circ\vec{i} + 4\text{Sin}60\vec{j} = 2\vec{i} + 2\sqrt{3}\vec{j}$.

$\Rightarrow \vec{R} = 6\vec{i} + 6\sqrt{3}\vec{j}$

$\therefore R = \sqrt{6^2 + (6\sqrt{3})^2} = \sqrt{36 + 108} = \sqrt{144} = 12$ N

(ii)



$x^2 + (\frac{1}{2}l) = l^2 \Rightarrow x = \frac{\sqrt{3}}{2}l$.

(letter $\underline{l}$)

6. contd..

Taking moments about a:

Moment of the sum = the sum of the moments

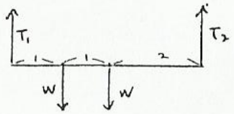
$12(x) = 8(0) + 8(\frac{\sqrt{3}}{2}l) + 4(0) \Rightarrow 12x = 4\sqrt{3}l$

$\Rightarrow x = \frac{\sqrt{3}}{3}l = \frac{1}{\sqrt{3}}l$ m.

The moment of the forces about a was $4\sqrt{3}l$ Nm, so the moment $M = 4\sqrt{3}l$ Nm.

EXERCISE 8.E

1.

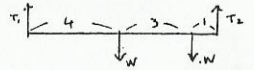


1: $T_1 + T_2 = 2W$

2: $W(1) + W(2) = T_2(4)$

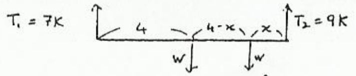
$\Rightarrow T_2 = \frac{3}{4}W \Rightarrow T_1 = 1\frac{1}{4}W$.

2.



1: $T_1 + T_2 = 2W$

2: $W(4) + W(7) = T_2(8) \Rightarrow T_2 = \frac{11}{8}W \Rightarrow T_1 = \frac{5}{8}W$.

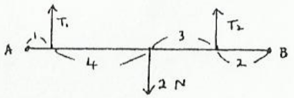


1. $7K + 9K = 2W \Rightarrow K = \frac{1}{8} \Rightarrow T_1 = \frac{7}{8}W, \quad T_2 = \frac{9}{8}W$

2. $W(4) + W(8-x) = \frac{9}{8}W(8)$

$\Rightarrow 12W - xW = 9W \Rightarrow x = 3$

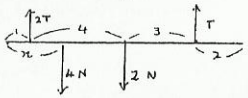
3.



1: $T_1 + T_2 = 2$

2: $2(5) = T_1(1) + T_2(8) \Rightarrow T_1 + T_2 = 10$

Solving these gives $T_1 = \frac{8}{7}N$, $T_2 = \frac{6}{7}N$

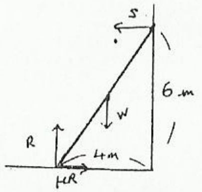


1: $T + 2T = 6 \Rightarrow T = 2$ N

2: $4(x) + 2(5) = 4(1) + 2(8)$

$\Rightarrow x = 2\frac{1}{2}$ cm.

4.



1. $R = W$
2. $\mu R = S$
3. $W(2) = S(6) \Rightarrow S = \frac{1}{3}W$.

2: $\mu W = \frac{1}{3}W \Rightarrow \mu = \frac{1}{3}$

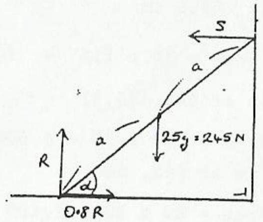
5. (i) Friction

(ii) Moment

/...

5. contd....

1. $R = 245$
2. $0.8R = S$
3. $245(a\cos\alpha) = S(2a\sin\alpha)$



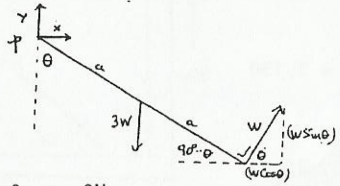
Eq.2 $\Rightarrow S = 0.8(245) = 196$

Eq. 3 $\Rightarrow 245\cos\alpha = (196)2\sin\alpha$

$\Rightarrow 245 = 392\tan\alpha$

$\Rightarrow \tan\alpha = \frac{245}{392} = \frac{5}{8}$

6. (i)



(ii) 1. $Y + W\sin\theta = 3W$

2. $X + W\cos\theta = 0$

3. (Taking moments about p, using unresolved forces)

$3W(a\sin\theta) = W(2a)$

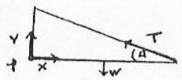
$\Rightarrow \sin\theta = 2/3 = 0.6667 \Rightarrow \theta = 41^\circ 49'$

(iii) $X = -W\cos\theta = -W(\frac{\sqrt{5}}{3}) = -\frac{\sqrt{5}}{3}W$.

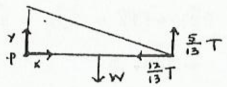
$Y = 3W - W\sin\theta = 3W - W(\frac{2}{3}) = \frac{7W}{3}$.

7. Since $\tan A = \frac{5}{12}$, $\cos A = \frac{12}{13}$, $\sin A = \frac{5}{13}$

Forces



Resolved



1. $Y + \frac{5}{13}T = W$
2. $X = \frac{12}{13}T$
3. Taking moments about p:

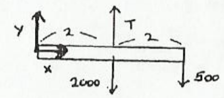
$W(6) = \frac{5}{13}T(12) \Rightarrow T = \frac{13}{10}W$

$\therefore X = \frac{12}{13}(\frac{13}{10}W) = \frac{6}{5}W$

$Y + \frac{5}{13}(\frac{13}{10}T) = W \Rightarrow Y = \frac{1}{2}W$

Ans: (i) $\frac{6}{5}W$, $\frac{1}{2}W$ (ii) $\frac{13}{10}W$

8. (a)



1: $T + Y = 2000 + 500 \Rightarrow T + Y = 2{,}500$

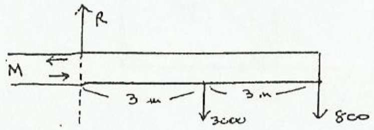
2: $X = 0$

3: $2000(2) + 500(4) = T(2) \Rightarrow T = 3000$ N $\therefore Y = 2500 - 3000 = -500$ N

8. contd ...

The reaction at the hinge has magnitude 500 N and is downwards.
The tension is 3000 N.

(b)



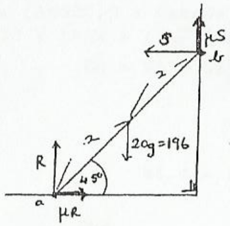
1: R = 3000 + 800 = 3800 N

2: None

3: The clockwise moments about the wall's end are 3000(3) + 800(6) = 13,800.

$\therefore$ M must be an anti-clockwise moment of magnitude 13,800 Nm.

9.



1: $R + \mu S = 196$

2: $\mu R = S$

3. Taking moments about $\underline{a}$,

$196(2\cos 45^{\circ}) = S(4\sin 45^{\circ}) + \mu S(4\cos 45^{\circ})$

But $\cos 45^{\circ} = \sin 45^{\circ} = 1/\sqrt{2}$

$\therefore\ 196(2) = S(4) + \mu S(4)$

$\Rightarrow S + \mu S = 98$

$\Rightarrow S(1+\mu) = 98$

$\Rightarrow \boxed{S = \dfrac{98}{1+\mu}}$

Now $\mu R = S \Rightarrow R = \dfrac{1}{\mu}S = \dfrac{1}{\mu}\left(\dfrac{98}{1+\mu}\right) = \dfrac{98}{\mu(1+\mu)}$

Putting these into equation 1, we get

$\dfrac{98}{\mu(1+\mu)} + \mu\left(\dfrac{98}{1+\mu}\right) = 196$. Multiply by $\mu(1+\mu)$

$\Rightarrow 98 + 98\mu^2 = 196\mu(1+\mu)$. Divide by 98.

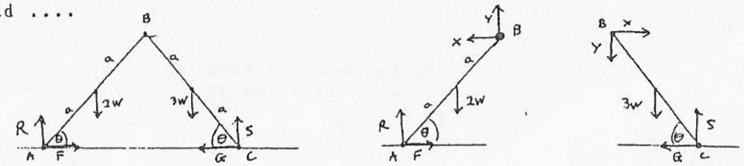
$\Rightarrow 1 + \mu^2 = 2\mu(1+\mu) \Rightarrow 1+\mu^2 = 2\mu + 2\mu^2$

$\Rightarrow \mu^2 + 2\mu - 1 = 0$

$\Rightarrow \mu = \dfrac{-2 \pm \sqrt{4+4}}{2} = \dfrac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$

Since $\mu > 0$, $\mu = \sqrt{2} - 1$ Q.E.D.

10. contd ....
(i)



(ii) (From system ABC)

1: $R + S = 2W + 3W \Rightarrow R + S = 5W$

2: $F = G$

3: Taking moments about $\underline{a}$.

$2W(a\cos\theta) + 3W(3a\cos\theta) = S(4a\cos\theta)$

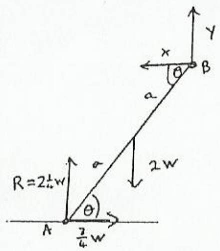
$\Rightarrow 4S = 11W \Rightarrow S = 2\tfrac{3}{4}W$.

$\therefore\ R = 2\tfrac{1}{4}W$, from equation 1.

(iii) Since $R < S$, $\mu R < \mu S$ $\therefore$ slipping will occur at A first.

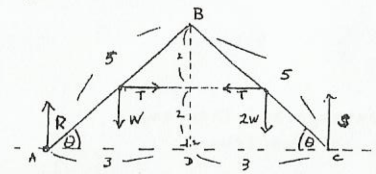
(iv) Let the rod AB be on the point of slipping.

$\therefore\ F = \mu R = \tfrac{1}{3}R = \tfrac{1}{3}(2\tfrac{1}{4}W) = \tfrac{3}{4}W$



1. $R + Y = 2W$
2. $\tfrac{3}{4}W = X$
3. Taking moments about B:
$2W(a\cos\theta) + \tfrac{3}{4}W(2a\sin\theta) = 2\tfrac{1}{4}(2a\cos\theta)$
$\Rightarrow 2\cos\theta + \tfrac{3}{2}\sin\theta = \tfrac{9}{2}\cos\theta$
$\Rightarrow 3\sin\theta = 5\cos\theta$
$\Rightarrow 3\tan\theta = 5$
$\Rightarrow \tan\theta = 5/3$
$\Rightarrow \theta = \tan^{-1} 5/3 = 59^{\circ}2'$

11.



Since $\cos\theta = \dfrac{3}{5}$, $\sin\theta = \dfrac{4}{5}$ and $\tan\theta = \dfrac{4}{3}$

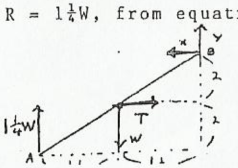
The system ABC

1. $R + S = W + 2W \Rightarrow R + S = 3W$
2. $T = T$
3. Taking moments about A:

$W(1\tfrac{1}{2}) + T(2) + 2W(4\tfrac{1}{2}) = T(2) + S(6)$

$\Rightarrow S = 1\tfrac{3}{4}W$

$\therefore\ R = 1\tfrac{1}{4}W$, from equation 1.



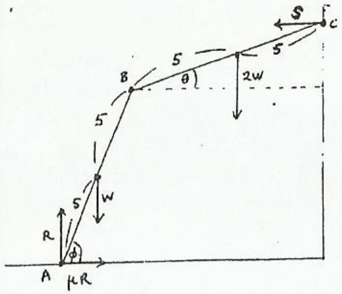
The rod AB

1. $1\tfrac{1}{4}W + Y = W \Rightarrow Y = -\tfrac{1}{4}W$
2. $T = X$

. contd .....

3. Taking moments about B.
$T(2) + W(1\frac{1}{2}) = 1\frac{1}{4}W(3)$
$\Rightarrow T = 1\frac{1}{8}W$

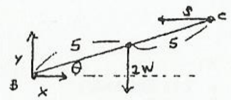
swer: $T = 1\frac{1}{8}W$, $R = 1\frac{1}{4}W$, $S = 1\frac{3}{4}W$



Since $\tan\phi = \frac{4}{3}$, $\sin\phi = \frac{4}{5}$, $\cos\phi = \frac{3}{5}$

1. $R = W + 2W = 3W$
2. $\mu R = S$
3. Taking moments about A:
$W(5\cos\phi) + 2W(10\cos\phi + 5\cos\theta) = S(10\sin\phi + 10\sin\theta.)$

$\Rightarrow 3W + 12W + 10W\cos\theta = 8S + 10S\sin\theta$
$\Rightarrow 15W + 10W\cos\theta = 8S + 10S\sin\theta$

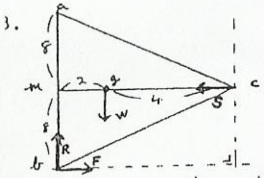


4. $Y = 2W$.
5. $X = S$.
6. Taking moments about B:
$2W(5\cos\theta) = S(10\sin\theta)$
$\Rightarrow 10W\cos\theta = 10S\sin\theta$

This means that equation3 reads:
$15W + 10S\sin\theta = 8S + 10S\sin\theta$
$\Rightarrow S = \frac{15}{8}W$

juation 2 now reads:
$R = S \Rightarrow \mu(3W) = \frac{15}{8}W \Rightarrow \mu = \frac{5}{8}$

3.



$|mc| = 6$ (from Pythagoras' Theorem)
The centroid, g, is 2 cm from m, 4 cm from c.

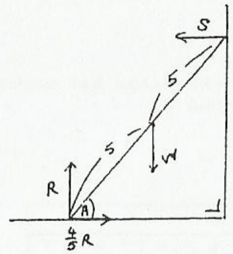
ssume it is on the point of slipping. Therefore, $F = \mu R$.

. $R = W$
. $\mu R = S$
. Taking moments about b:
$W(2) = S(8) \Rightarrow S = \frac{1}{4}W$.

quation 2 $\Rightarrow \mu R = S \Rightarrow \mu(W) = \frac{1}{4}W \Rightarrow \mu = \frac{1}{4}$

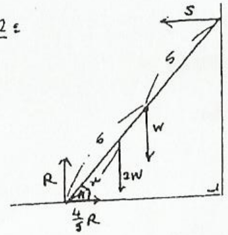
he least value of is $\frac{1}{4}$.

14. Part 1:



1. $R = W$
2. $\frac{4}{5}R = S$
3. Taking moments about the foot of the ladder:
$W(5\cos A) = S(10\sin A) \Rightarrow W\cos A = 2S\sin A$
But $S = \frac{4}{5}R = \frac{4}{5}W$.
$\therefore W\cos A = 2(\frac{4}{5}W)\sin A$
$\Rightarrow \cos A = \frac{8}{5}\sin A \Rightarrow 1 = \frac{8}{5}\tan A \Rightarrow \tan A = \frac{5}{8}$

Part 2:

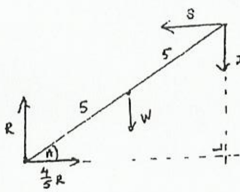


Since $\tan A = \frac{3}{4}$, $\cos A = \frac{4}{5}$, $\sin A = \frac{3}{5}$

1: $R = 2W + W = 3W$
2: $\frac{4}{5}R = S \Rightarrow S = \frac{4}{5}(3W) = \frac{12}{5}W$
3: $2W(x\cos A) + W(5\cos A) = S(10\sin A)$
$\Rightarrow 2W(\frac{4}{5}x) + W(4) = S(6)$
$\Rightarrow \frac{8}{5}xW + 4W = 6S$

But $S = \frac{12}{5}W \therefore \frac{8}{5}xW + 4W = 6(\frac{12}{5}W)$
$\Rightarrow 8x + 20 = 72 \Rightarrow x = 6.5$m

Part 3:



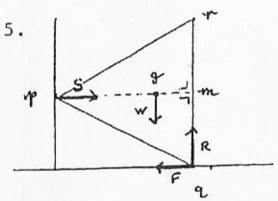
Assume it is just on the point of slipping when the man reaches the top.

1: $R = W + 2W = 3W$
2: $\frac{4}{5}R = S \Rightarrow S = \frac{4}{5}(3W) = \frac{12}{5}W$

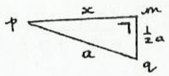
3: $W(5\cos A) + 2W(10\cos A) = S(10\sin A)$
$\Rightarrow 5W = 2S\tan A$

But $S = \frac{12}{5}W$, $\therefore 5W = \frac{24}{5}W\tan A \Rightarrow \tan A = \frac{25}{24}$

15.



Let $|pm| = x$.



$x^2 + (\frac{1}{2}a)^2 = a^2 \Rightarrow x = \frac{\sqrt{3}}{2}a$
$\therefore |pg| = \frac{2}{3}(\frac{\sqrt{3}}{2}a) = \frac{1}{\sqrt{3}}a$
$|gm| = \frac{1}{3}(\frac{\sqrt{3}}{2}a) = \frac{1}{2\sqrt{3}}a$

1. $R = W$
2. $F = S$
3. $S(\frac{1}{2}a) = W(\frac{1}{2\sqrt{3}}a) \Rightarrow S = \frac{1}{\sqrt{3}}W$

The reactions are $\frac{1}{\sqrt{3}}W$, W

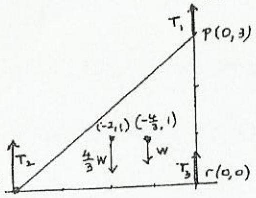
If it is on the point of slipping, then $F = \mu R$.

5. contd ...

quation 2 $\Rightarrow \mu R = S \Rightarrow \mu W = \frac{1}{\sqrt{3}}W \Rightarrow \mu = \frac{1}{\sqrt{3}}$

$\therefore$ The least value of μ is $\frac{1}{\sqrt{3}}$.

6.



Centroid is at $\left(\frac{0+0+\bar{4}}{3}, \frac{0+3+0}{3}\right)$

$= \left(\frac{-4}{3}, 1\right)$

$\therefore$ $T_1 + T_2 + T_3 = \frac{4}{3}W + W \Rightarrow T_1 + T_2 + T_3 = \frac{7}{3}W$

$\therefore$ Taking moments about the y-axis:

$T_2(4) + T_1(0) + T_3(0) = \frac{4}{3}W(2) + W(\frac{4}{3})$

$\Rightarrow 4T_2 = 4W \Rightarrow T_2 = W$

3: Taking moments about the x-axis:

$T_1(3) + T_2(0) + T_3(0) = \frac{4}{3}W(1) + W(1) \Rightarrow T_1 = \frac{7}{9}W$

But $T_1 + T_2 + T_3 = \frac{7}{3}W \Rightarrow T_3 = \frac{5}{9}W$. Answer: $\frac{7}{9}W$, W, $\frac{5}{9}W$

EXERCISE 8.F
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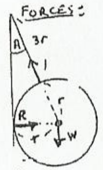
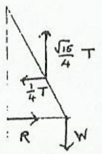
1.



Sin A $= \frac{r}{3r+r} = \frac{1}{4}$

$\therefore$ Cos A $= \frac{\sqrt{15}}{4}$

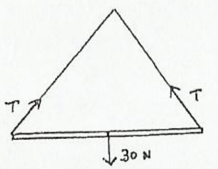
Resolved Forces:



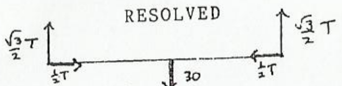
1. $\frac{\sqrt{15}}{4}T = W \Rightarrow T = \frac{4W}{\sqrt{15}}$

2. $R = \frac{1}{4}T = \frac{1}{4}(\frac{4W}{\sqrt{15}}) = W/\sqrt{15}$

2. FORCES



RESOLVED



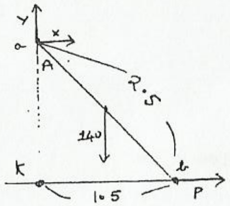
1: $\frac{\sqrt{3}}{2}T + \frac{\sqrt{3}}{2}T = 30$

$\Rightarrow T = \frac{30}{\sqrt{3}} = 10\sqrt{3}$ N.

3. (i) "....... are concurrent"

3. contd ...

(ii)



$|ak|^2 + |kb|^2 = |ap|^2$

$\Rightarrow |ak|^2 + 2.25 = 6.25$

$\Rightarrow |ak| = 2$

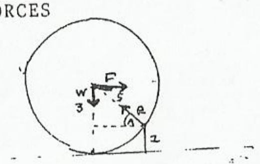
1. $Y = 240$
2. $P + X = 0 \Rightarrow X = -P$
3. Taking moments about $\underline{a}$:

$240(0.75) = P(2) \Rightarrow P = 90$ N

$\vec{X} = -90\vec{i}$, $\vec{Y} = 240\vec{j}$

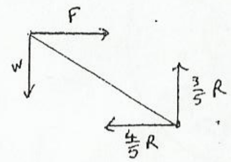
$\therefore$ Resultant $= -90\vec{i} + 240\vec{j} \Rightarrow |\vec{R}| = \sqrt{(-90)^2 + (240)^2} = 256$ N

4. (i) FORCES



Since Sin$A = \frac{3}{5}$, Cos$A = \frac{4}{5}$

Resolved Forces

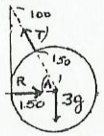


(ii) In accordance with Theorem 8.7

(iii) 1. $\frac{3}{5}R = W \Rightarrow R = \frac{5}{3}W$

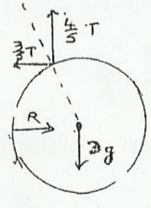
2. $F = \frac{4}{5}R = \frac{4}{5}(\frac{5}{3}W) = \frac{4}{3}W$

5. (a) Forces



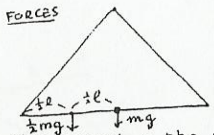
Cos$A = \frac{150}{250} = \frac{3}{5}$ $\therefore$ Sin $A = \frac{4}{5}$

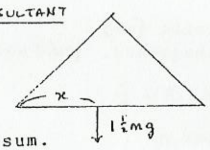
Resolved Forces



1: $\frac{4}{5}T = 3g \Rightarrow T = \frac{15}{4}g$.

(b) Step 1. To find centre of gravity:

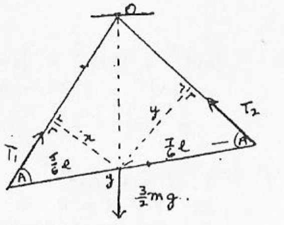




The sum of the moment = the moment of the sum.

$\frac{1}{2}mg(\frac{1}{2}\ell) + mg(\ell) = 1\frac{1}{2}mg(x) \Rightarrow x = \frac{5}{6}\ell$

5. contd ...



The resultant weight, $\frac{3}{2}mg$ acts through g, which must be below o in accordance with Theorem 8.7

Taking moments about g:

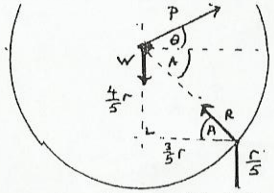
$T_1(x) = T_2(y)$

But $x = \frac{5}{6}l\,\text{Sin}A$, $y = \frac{7}{6}l\,\text{Sin}A$

$\therefore T_1(\frac{5}{6}l\,\text{Sin}A) = T_2(\frac{7}{6}l\,\text{Sin}A)$

$\Rightarrow \frac{T_1}{T_2} = \frac{7}{5}$.

6.



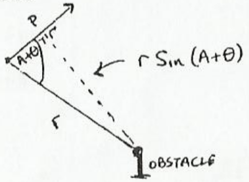
$\text{Sin}A = \frac{4}{5}$

$\therefore \text{Cos}\,A = \frac{3}{5}$

Taking moments about the top of the obstacle

$W(\frac{3}{5}r) = P(r\text{Sin}(A+\theta))$

$\Rightarrow P = \frac{3W}{5\text{Sin}(A+\theta)}$.



(i) In this case $\theta = 0$

$\therefore P = \frac{3W}{5\text{Sin}A} = \frac{3W}{4}$

(ii) In this case we want a minimum value for $\frac{3W}{5\text{Sin}(A+\theta)}$ This value is attained when

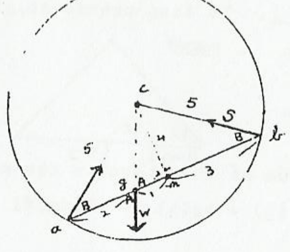
$\text{Sin}(A+\theta) = 1$, and is $\frac{3W}{5}$

7.

Let m be the midpoint [ab].

$|cm| = 4$ (by Pythagoras), $|gm| = 1$ and $|gc| = \sqrt{17}$.

$\text{Sin } A = \frac{4}{\sqrt{17}}$ and $\text{Sin } B = \frac{4}{5}$



Taking moments about a:

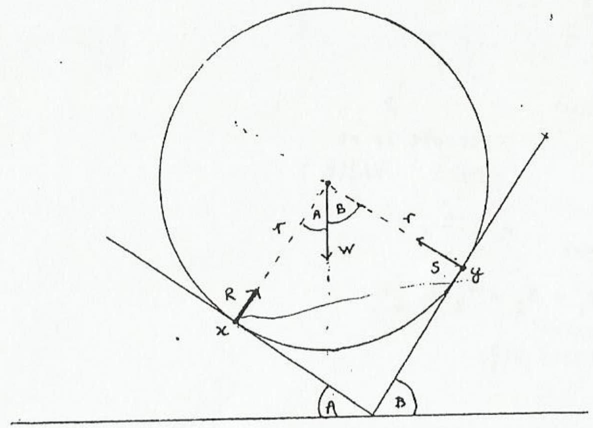
$W(2\text{Sin}A) = S(6\text{Sin}B)$

$\Rightarrow W(\frac{8}{\sqrt{17}}) = S(\frac{24}{5}) \Rightarrow S = \frac{5W}{3\sqrt{17}}$

Taking moments about b:

$W(4\text{Sin}A) = R(6\sin B) \Rightarrow W(\frac{16}{\sqrt{17}}) = R(\frac{24}{5}) \Rightarrow R = \frac{10W}{3\sqrt{17}}$

8.



Since $\text{Tan A} = \frac{1}{2}$, $\text{Sin A} = \frac{1}{\sqrt{5}}$ and $\text{Cos A} = \frac{2}{\sqrt{5}}$.

Since $\text{Tan B} = \frac{3}{4}$, $\text{Sin B} = \frac{3}{5}$ and $\text{Cos B} = \frac{4}{5}$.

Also, $\text{Sin}(A + B) = \text{Sin A Cos B} + \text{Cos A Sin B} = (\frac{1}{\sqrt{5}})(\frac{4}{5}) + (\frac{2}{\sqrt{5}})(\frac{3}{5}) = \frac{10}{5\sqrt{5}} = \frac{2}{\sqrt{5}}$.

Taking moments about x,

$W(r\text{ Sin A}) = S(r\text{ Sin}(A + B))$

$\Rightarrow W(\frac{1}{\sqrt{5}}r) = S(\frac{2}{\sqrt{5}}r)$

$\Rightarrow S = \frac{1}{2}W$. Q.E.D.

CHAPTER 9. HYDROSTATICS.

EXERCISE 9.A

1. $0.83 \times 1000 = 830 \text{ kg/m}^3$

2. $\frac{375}{1000} = 0.375$

3. (i) 1 m^3 has mass $13.6 \times 1{,}000{,}000 = 13{,}600{,}000$ grammes $= 13{,}600$ kg.

$\therefore$ Density $= 13{,}600 \text{ kg/m}^3$.

(ii) $\frac{13{,}600}{1{,}000} = 13.6$

4. Volume $= 0.1 \times 0.06 \times 0.02 = 0.00012 \text{ m}^3$.

Density $= \frac{\text{mass}}{\text{volume}} = \frac{1.08}{0.00012} = 9000 \text{ kg/m}^3$.

Relative density $= \frac{9000}{1000} = 9$.

5. Volume $= \frac{\text{mass}}{\text{density}} = \frac{70}{980} = \frac{1}{14} \text{ m}^3$.

Mass = Volume x Density $= \frac{1}{14} \times 2450 = 175$kg.

6.

s	v	sv
1	60	60
0.95	40	38
	100	98

$\bar{s} = \frac{98}{100} = 0.98$

7.

s	v	sv
7	10	70
9	1	9
	11	79

$\bar{s} = \frac{79}{11} = 7.182$

8.

s	v	sv
1	100	100
0.9	x	0.9x
	100+x	100+0.9x

$\frac{100 + 0.9x}{100 + x} = 0.9625$

$\Rightarrow 100 + 0.9x = 96.25 + 0.9625x$

$\Rightarrow x = 60$ ml

9. Volume $= 0.4 \times 0.3 \times 0.2 = 0.024 \text{ m}^3$

Weight $= V\rho g = (0.024)(1000g) = 24g$ N

10. Volume $= \pi r^2 h = \pi (0.03)^2 (0.1) = 0.00009\pi \text{ m}^3$

Weight $= V\rho g = (0.00009\pi)(800)g = 0.072\pi g$ N

11.
(a) $V = \frac{1}{3}\pi h(R^2 + Rr + r^2) = \frac{1}{3}\pi(18)(81 + 54 + 36) = 1026\pi \text{ cm}^3 = 0.001026\pi \text{ m}^3$

Weight $= V\rho g = (0.001026\pi)(1000)g = 1.026\pi g$ N

(b) $V = \frac{1}{3}\pi h(R^2 + Rr + r^2) = \frac{1}{3}\pi(6)(49 + 42 + 36) = 254\pi \text{ cm}^3 = 0.000254\pi \text{ m}^3$

Weight $= V\rho g = (0.000254\pi)(950)(g) = 0.2413\pi g$ N

(c) Density $= \frac{\text{mass}}{\text{volume}} = \frac{1.026\pi + 0.2413\pi}{0.001026\pi + 0.000254\pi} = \frac{1.2673}{0.00128} = 990$

Specific gravity $= \frac{990}{1{,}000} = 0.99$

EXERCISE 9.B

1. (i) Pressure $= h\rho g = (2)(1000)g = 2000g \text{ N/m}^2$

(ii) Thrust = Pressure x Area $= 2000g \times (2)^2 = 8000g$ N

Weight $= V\rho g = (2)^3(1000)g = 8000g$ N

2. (i) Pressure $= h\rho g = (0.11)(850)g = 93.5g \text{ N/m}^2$

(ii) Thrust = Pressure x Area $= (93.5g)\pi(0.05)^2 = 0.23375\pi g$ N

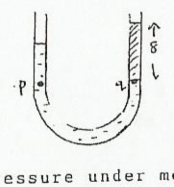
(iii) Weight $= V\rho g = \pi r^2 h\rho g = \pi(0.05)^2(0.11)(850)g = 0.23375\pi g$ N

3. Thrust = Pressure x Area $= (h\rho g)(\pi r^2) = (0.1)\rho g(\pi(0.2)^2) = 0.00004\pi\rho g$

Weight $= V\rho g = \frac{1}{3}\pi h(R^2+Rr+r^2)\rho g = \frac{1}{3}\pi(0.1)\{(0.05)^2 + (0.05)(0.02) + (0.02)^2\}\rho g = 0.00013\pi\rho g$

The ratio is, therefore, 4 : 13

4. (a)



Pressure at p = Pressure at q

$\Rightarrow h(1000)g = 8(850)g$

$\Rightarrow h = 6.8$cm

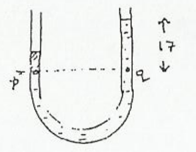
$\therefore$ Difference $= 8 - 6.8 = 1.2$ cm

(b) Pressure under mercury = Pressure under oil

$\Rightarrow h(13600)g = 8(850)g$

$\Rightarrow 13600h = 6800 \Rightarrow h = 0.5$cm

5. (a)



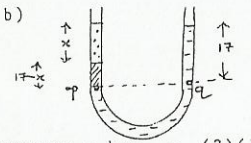
Pressure at p = Pressure at q.

$\Rightarrow h = (13600)g = 17(1000)g$

$\Rightarrow h = 1.25$ cm

$\therefore$ Difference = 17 - 1.25 = 15.75cm

(b)



Pressure at p = Pressure at q

$\Rightarrow x(850)g + (17-x)(13600)g = 17(1000)g$

$\Rightarrow x = 16.8$ cm

6. Pressure = $h\rho g$ = $(3)(1000)g = 3000g$ N/m^2.
Thrust = Pressure x Area = $(3000g)(2 \times 2) = 12000g$ N

(i) $2 \times 2 \times h = 1 \times 1 \times 1 \Rightarrow h = \frac{1}{4}$m.

(ii) $P = h\rho g = (\frac{1}{4})(1000)(g) \Rightarrow 250g$ N/m^2

(iii) $T = P \times A = (250g)(4) = 1000g$ N

7. (i) $\pi R^2 h = \frac{4}{3}\pi r^3$

$\Rightarrow \pi(16)h = \frac{4}{3}\pi(27) \Rightarrow h = 2\frac{1}{4}$cm = 0.0225m

(ii) $P = h\rho g = (0.0225)(1000)g = 22.5g$ N/m^2

(iii) Thrust = $P \times A = (22.5g)(\pi(16)) = 360\pi$ N

8. (i) $\pi R^2 h = \frac{4}{3}\pi r^3$

$(36)h = \frac{4}{3}\pi(27) \Rightarrow h = 1$ cm = 0.01 m

(ii) $P = h\rho g = (0.01)(900)g = 9g =$ N/m^2

(iii) $T = P \times A = (9g)(\pi(0.06)^2) = 0.0324\pi g$ N

9. $1 \frac{\text{dyne}}{\text{cm}^2} = \frac{10^{-5} \text{ Newtons}}{10^{-4} \text{ m}^2} = 0.1$ N/m^2

10. Thrust = Pressure x Area = $(h\rho g)(\pi r^2)$
$= (2)(1250)(9.8)(\frac{22}{7})(1.4)^2 = 150920$ N = 150.92 kN

Volume = $\frac{M}{\rho} = \frac{750}{2{,}500} = 0.3$m^3

Let x = Increase in depth $\Rightarrow \pi r^2 x = 0.3 \Rightarrow x = \frac{0.3}{\pi r^2}$

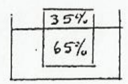
$\therefore$ Increase in pressure = $h\rho g = \frac{0.3}{\pi r^2}(1250)g$

$\therefore$ Increase in thrust = $P \times A = \frac{0.3}{\pi r^2}(1250)g(\pi r^2) = 3675$ N
= 3.675 kN

EXERCISE 9.C

1. Bouyancy = 12 - 8 = 4
$B = \frac{W}{s} \Rightarrow 4 = \frac{12}{s} \Rightarrow s = 3$

2.



$W = V\rho g$. $B = (\frac{65}{100}V)(1000)g = 650Vg$

But $B = W \Rightarrow \rho = 650 \Rightarrow s = 0.65$

3. 0.9, as in the last question

4. B = 18 - 15 = 3
$B = \frac{W}{s} \Rightarrow 3 = \frac{18}{s} \Rightarrow s = 6$

$B_L = 18 - 16 = 2$. $B_L = s_L B_w \Rightarrow 2 = s_L(3) \Rightarrow s_L = \frac{2}{3}$

5. (i) $B = \frac{W}{s} = \frac{30}{6} = 5$ $\therefore$ Apparent weight = 30 - 5 = 25 N

(ii) $B_L = s_L B_w = (0.9)(5) = 4.5$ $\therefore$ Apparent weight = 30 - 4.5 = 25.5 N

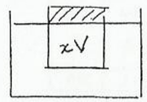
6. $B_L = s_L B_w = S_L(\frac{W}{s}) = 13.6(\frac{200}{17}) = 160$
Apparent weight = 200 - 160 = 40 N

7. $s = \frac{3}{4}$, as in question 2. $\therefore \rho = \frac{3}{4} \times 1000 = 750$.
$W = V\rho g = (0.1)(750)g = 75g$
$B = \frac{W}{s} = \frac{75g}{3/4} = 100g$

$B = W + T \Rightarrow 100g = 75g + T \Rightarrow T = 25g$

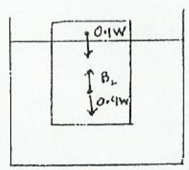
8. Bouyancy in water, $B_w = 80 - 60 = 20$.
Bouyancy in oil, $B_L = 80 - 64 = 16$.
$B_L = s_L B_w \Rightarrow 16 = s_L(20) \Rightarrow s_L = 0.8$.

9. It's specific gravity is 0.8, as in question 2
Let V = it's volume, xV = volume under the liquid



$W = V\rho g = V(800)g = 800Vg$
$B = xV(1200)g = 1200xVg$
Since it is in equilibrium, $W = B \Rightarrow 800Vg = 1200xVg \Rightarrow x = \frac{2}{3}$
$\therefore$ $\frac{1}{3}$ of its mass, or $33\frac{1}{3}$%, is above the surface.

10. (i)



$B_L = s_L B_w = s_L(\frac{W}{s})$ where W = weight of the <u>immersed</u> part.

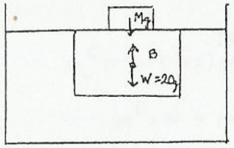
$\therefore B_L = (1.1)(\frac{0.9W}{s}) = \frac{0.99W}{s}$

Since it is in equilibrium $0.1W + 0.9W = B_L$

$\Rightarrow W = \frac{0.99W}{s} \Rightarrow s = 0.99$

(ii) 0.99 of its mass will be below, as in question 2.

11. It's specific gravity is $\frac{3}{4}$, as in question 2.



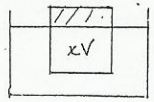
Let M = the mass of the glass

$$B_L = s_L B_w = s_L(\frac{W}{s}) = (0.8)(\frac{20g}{0.75}) = 21\frac{1}{3}g$$

$$B = Mg + 20g \Rightarrow 21\frac{1}{3}g = Mg + 20g \Rightarrow M = 1\frac{1}{3}kg$$

12. $B = 360 - 330 = 30$.
$B = \frac{W}{s} \Rightarrow 30 = \frac{360}{s} \Rightarrow s = 12$. It is lead.

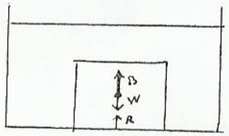
13. (a)



Let V = its volume, xV = volume under the sea.

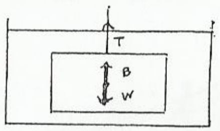
B = weight of liquid displaced = (xV)(1030)g.
W = weight of the object = V(900)g.
Since B = W, $xV(1030)g = V(900g) \Rightarrow x = 0.87$.
Answer: 87%

(b) $V = (0.8)(0.6)(0.4) = 0.192\ m^3$



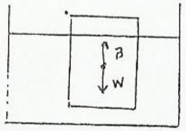
$W = V\rho g = (0.192)(2500)g = 480g$
$B = (0.192)(1000)g = 192g$
$B + R = W \Rightarrow 192g + R = 480g$
$\Rightarrow R = 288g = 2822.4\ N$

14. (a) (i) $V = \frac{M}{\rho} = \frac{12.5}{2500} = 0.005\ m^3$



(ii) $M = V\rho = 0.005(800) = 4kg$
(iii) $W = 12.5g$, $B = 4g$
$W = T + B \Rightarrow T = 12.5 - 4 = 8.5g = 83.3\ N$

(b)
(i) $V = (0.1)(0.2)(0.08) = 0.0016\ m^3$
(ii)



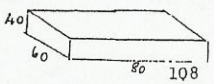
$M = V\rho \Rightarrow 1 = 0.0016\rho \Rightarrow \rho = 625$
$\therefore s = \frac{625}{1000} = \frac{5}{8}$

$\frac{5}{8}$th of it's length is submerged

$\therefore$ Depth $= \frac{5}{8} \times 20 = 12.5$ cm

15. $B = 40 - 35 = 5$ $B = \frac{W}{s} \Rightarrow 5 = \frac{40}{s} \Rightarrow s = 8$

Total area $= (80 \times 60) + 2(80 \times 40) + 2(60 \times 40) = 16000\ cm^2 = 1.6\ m^2$



15. contd ...

Volume $= 1.6 \times .0015 = 0.0024\ m^3$.
Weight $= V\rho g = (0.0024)(8000)g = 19.2g$.
Let X = the depth of the tank in the lake.
B = weight of liquid displaced $= (0.6 \times 0.8 \times X)(1000)g = 480\ Xg$.
$B = W \Rightarrow 480\ Xg = 19.2g = 0.04m = 4cm$.

16. Pressure at 5 cm $= h\rho g = (0.05)(900)g = 45g$
Pressure at 9.5 cm $= (0.05)(900)g + (0.045)1000)g = 90g$.
$\therefore$ Pressure at 9.5 cm = Twice pressure at 5cm

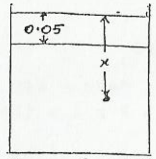


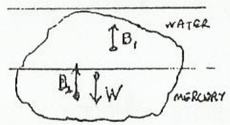
Diagram for part 3

Pressure at 3 cm $= (0.03)(900)g = 27g$
5 times pressure at 3cm = 135g
Lex x = the depth (in metres)

Pressure $= (0.05)(900)g + (x-0.05)(1000)g = 135g \Rightarrow 45g + 1000g$
$- 50g = 135g \Rightarrow x = 0.14\ m = 14cm$

17. (a). Let A = Athmospheric pressure
Pressure at 14m = 2 x Pressure at 2m.
$A + 14(1000)g = 2(A + 2(1000)g) \Rightarrow A = 10000g = 98000\ N/m^2$

(b)



$\therefore W = V\rho g = (v_1 + v_2)(7800)g = 7800(v_1+v_2)g$
$B_1 = v_1(1000)g = 1000v_1\ g$
$B_2 = v_2(13,600)g = 13,600v_2\ g$

Since the body is in equilibrium $B_1 + B_2 = W$
$\Rightarrow 1000v_1\ g + 13,600v_2\ g = 7800(v_1 + v_2)g$
$\Rightarrow 5,800v_2 = 6,800v_1 \Rightarrow \frac{v_1}{v_2} = \frac{58}{68} = \frac{29}{34}$

18. (i) Bouyancy = weight of liquid displaced
$= (\frac{4}{3}\pi(1)^3)(1000)g = \frac{2000}{3}\pi g$ N

(ii) Pressure $= h\rho g = (2)(1000)g = 2000g$
Thrust $= P \times A = (2000g)(\pi(1)^2) = 2000\pi g$ N

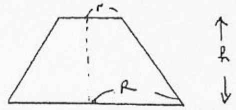
(iii) $B = F_u - F_d \Rightarrow \frac{2000}{3}\pi g = 2000\pi g - Fd \Rightarrow Fd = \frac{4000}{3}\pi g$ N

19. (i) Bouyancy = weight of displaced liquid $= (\frac{1}{3}\pi(2)^2(6))(900)g = 7200\pi g$ N

(ii) Pressure $= h\rho g = (7)(900)g = 6300g$ N
Thrust $= P \times A = (6300g)(\pi(2)^2) = 25,200\pi g$ N.

(iii) $B = F_u - F_d \Rightarrow 7200\pi g = 25,200\pi g - Fd$
$\Rightarrow Fd = 18,000\pi g$ N

20. (a)



Since $\pi r^2 = \frac{1}{4}(\pi R^2)$, $R = 2r$

Volume $= \frac{1}{3}\pi h \{ (2r)^2 + (2r)r + r^2 \}$

$= \frac{7}{3}\pi h r^2$

Weight $= V\rho g = \frac{7}{3}\pi h r^2 \rho g$

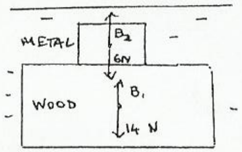
Pressure at base $= h\rho g$

$\therefore$ Thrust $= P \times A = (h\rho g)(\pi(2r)^2) = 4\pi h r^2 \rho g$

Ratio, Thrust : Weight $= 4\pi h r^2 \rho g : \frac{7}{3}\pi h r^2 \rho g = 12:7$

All surds are absent - how absurd!

(b)



$B_1 = \frac{W}{s} = \frac{14}{s}$

$B_2 = \frac{W}{s} = \frac{6}{10} = 0.6$

Apparent weight = Weight - Bouyancy.

$1.9 = 20 - (\frac{14}{s} + 0.6) \Rightarrow s = 0.8$

21. (a)

(i) $P = h\rho g = (\frac{1}{2})(1000)g = 500g$ N/m^2

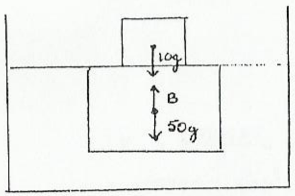
(ii) $T = P \times A = (500g)(\pi(\frac{1}{8})^2) = \frac{125}{16}\pi g$ N

(iii) $W = V\rho g = \frac{1}{3}\pi(\frac{1}{2})\{(\frac{1}{4})^2 + (\frac{1}{4})(\frac{1}{8}) + (\frac{1}{8})^2\}(1000)g = \frac{875}{16}\pi g$

$\therefore \frac{\text{Weight}}{T} = \frac{7}{1}$

(b) Specific gravity of the wood $= \frac{3}{4} = 0.75$ $\therefore$ Volume $= \frac{M}{\rho} = \frac{50}{750}$

$= \frac{1}{15}$m^3



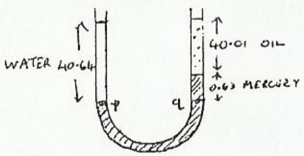
$B = 50g + 10g = 60g =$ Weight of liquid

$\therefore 60g = (\frac{1}{15})\rho g$

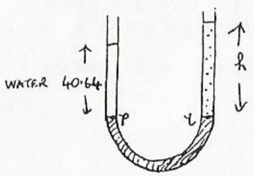
$\Rightarrow \rho = 60 \times 15 = 900$

$\Rightarrow s = 0.9$

22. (a) Part I,



Part 2



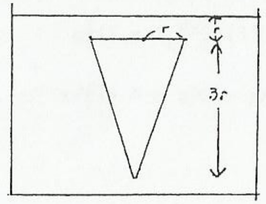
Pressure at p = Pressure at q.

$\therefore (40.64)(1000)g = (40.01)\rho g = (0.63)(13600)g \Rightarrow \rho = 801.6$

$(40.64)(1000g) = h(801.6)g \Rightarrow h = 50.7$ cm $= 507$mm.

22. contd ...

(b)



(i) Pressure $= h\rho g = r\rho g$

Thrust $= P \times A = r\rho g(\pi r^2) = \pi r^3 \rho g$

(ii) $B = V\rho g = (\frac{1}{3}\pi r^2(3r))\rho g = \pi r^3 \rho g$

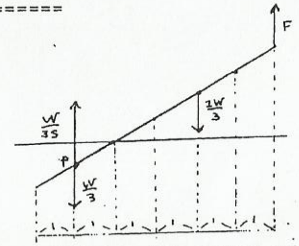
$B = Fu - Fd$

$\pi r^3 \rho g = Fu - \pi r^3 \rho g$

$\Rightarrow Fu = 2\pi r^3 \rho g.$

EXERCISE 9.D

1.



1: $F + \frac{W}{3S} = W$

2: Taking moments about p:

$\frac{2W}{3}(3) = F(5) \Rightarrow F = \frac{2}{5}W.$

$\therefore$ 1: $\Rightarrow \frac{2}{5}W + \frac{W}{3S} = W$

$\Rightarrow \frac{W}{3S} = \frac{3}{5}W \Rightarrow s = \frac{5}{9}$

2. (a) Let w = the weight of the body. Let B = Bouyancy in water

$\therefore w_1 = w - B.$

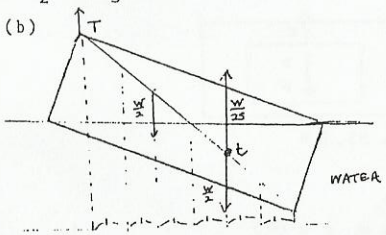
$B_L = s_L\, B_w \Rightarrow$ Bouyancy in liquid 1 = (0.8)B

$\therefore w_2 = w - 0.8B$

Similarily, $w_3 = w - 0.75B$

$5w_2 - 4w_3 = 5(w - 0.8B) - 4(w - 0.75B) = w - B = w_1$ q.e.d.

(b)



1. $T + \frac{W}{2S} = W$

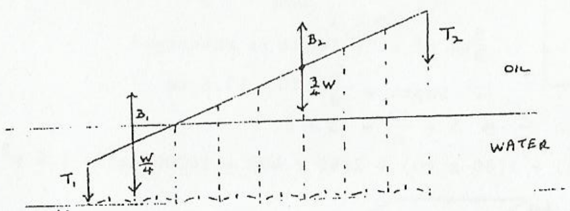
2. (Taking moments about t)

$\frac{W}{2}(2) = T(4) \Rightarrow T = \frac{1}{4}W$

$1 \Rightarrow \frac{1}{4}W + \frac{W}{2S} = W.$

$\Rightarrow \frac{W}{2S} = \frac{3}{4}W \Rightarrow s = \frac{2}{3}$

3.



$B_w = \frac{W}{s} \Rightarrow B_1 = \frac{\frac{W}{4}}{\frac{3}{4}} = \frac{W}{3}$

3. contd ...

$$B_L = s_L B_w = s_L\left(\frac{W}{s}\right) \Rightarrow B_2 = (0.9)\left(\frac{\frac{3}{4}W}{\frac{3}{4}}\right) = \frac{9W}{10}$$

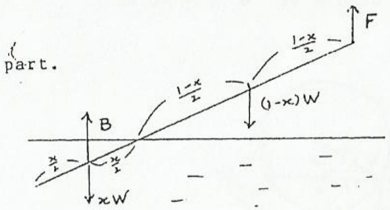
1. $\frac{W}{3} + \frac{9W}{10} = T_1 + \frac{W}{4} + \frac{3}{4}W + T_2 \Rightarrow T_1 + T_2 = \frac{7W}{30}$

2. (Taking moments about the lower end)

$$\frac{W}{3}(1) + \frac{9W}{10}(5) = \frac{W}{4}(1) + \tfrac{3}{4}W(5) + T_2(8)$$

$$\Rightarrow T_2 = \frac{5W}{48} \Rightarrow T_1 = \frac{7W}{30} - \frac{5W}{48} = \frac{31W}{240}$$

4. Let W = the weight of the rod.
Let x = the length of the submerged part.



$$B_w = \frac{W}{s} \Rightarrow B = \frac{xW}{0.36} = \frac{25xW}{9}$$

1: $\frac{25xW}{9} + F = W$

2: (Taking moments about p),

$$(1-x)\,W\left(\frac{x}{2} + \frac{1-x}{2}\right) = F\left(\frac{x}{2} + 1 - x\right)$$

$$\Rightarrow (1-x)W\left(\tfrac{1}{2}\right) = F\left(1 - \frac{x}{2}\right) \Rightarrow F = \frac{(1-x)W}{2-x}$$

Putting this result into equation 1 gives:

$$\frac{25xW}{9} + \frac{(1-x)W}{(2-x)} = W. \text{ Multiply by } \frac{9(2-x)}{W}$$

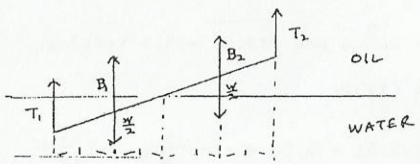
$\Rightarrow 25x(2-x) + 9(1-x) = 9(2-x)$

$\Rightarrow 25x^2 - 50x + 9 = 0 \Rightarrow (5x-1)(5x-9) = 0$

$\Rightarrow x = \frac{1}{5}$m ($x = \frac{9}{5}$m is too long)

Ans: 20 cm is submerged.

5.



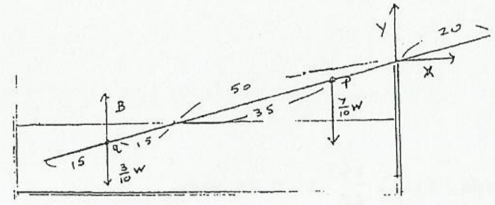
$$B_w = \frac{W}{s} \Rightarrow B_1 = \frac{\frac{1}{2}W}{6} = \frac{W}{12}, \quad B_L = s_L B_w = s_L\left(\frac{W}{s}\right) = 0.8\left(\frac{\frac{1}{2}W}{6}\right) = \frac{W}{15}$$

1: $T_1 + \frac{W}{12} + \frac{W}{15} + T_2 = W \Rightarrow T_1 + T_2 = \frac{51}{60}W$

2: (Taking moments about the lower end).

$$\frac{W}{12}(1) + \frac{W}{15}(3) + T_2(4) = \frac{W}{2}(1) + \frac{W}{2}(3) \Rightarrow T_2 = \frac{103}{240}W \Rightarrow T_1 = \frac{51}{60}W - \frac{103}{240}W = \frac{101}{240}W.$$

6.



Let X and Y be the horizontal and vertical components of the reaction at p. X = 0 since no other forces act along the vertical. Therefore, the reaction at p is vertical. We shall henceforth call it R.

$$B_w = \frac{W}{s} \Rightarrow B = \frac{\frac{3}{10}W}{s} = \frac{3W}{10s}$$

1: $\frac{3W}{10s} + R = W$

2: (Taking moments about q)

$$\frac{7}{10}W(50) = R(65) \Rightarrow R = \frac{7}{13}W$$

Putting this into equation 1 gives:

$$\frac{3W}{10s} + \frac{7}{13}W = W \Rightarrow \frac{3W}{10s} = \frac{6W}{13} \Rightarrow s = \frac{39}{60} = \frac{13}{20}.$$

CHAPTER 10. MOTION IN A CIRCLE.

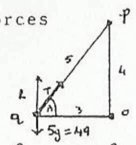
Exercise 10A

1. (i) $\frac{40}{20} = 2$ rads. (ii) $\frac{100}{20} = 5$ rads (iii) $\frac{15}{20} = 0.75$ rads
   (iv) $\frac{11}{20} = 0.55$ rads.
2. (i) $5 \times 3 = 15$cm (ii) $5 \times 0.8 = 4$cm (iii) $5 \times 1.2 = 6$cm
   (iv) $5 \times 1.7 = 8.5$cm.
3. $r = \frac{14}{3.5} = 4$ cm.
4. $r = \frac{2.07}{1.8} = 1.1\dot{5}$ cm.
5. Angular speed $= \frac{18}{10} = 1.8$ rads/sec.
6. $r = \frac{7}{4} = 1.75$cm.
7. In 60 seconds it turns 45 times.
   In 1 second it turns $\frac{3}{4}$ times $= \frac{3}{4}(2\pi)$rads/sec $= 4.7$rads/sec.
8. In 1 sec. it turns through 10 radians.
   In 60 secs. it turns through 600 radians.
   600 radians $= \frac{600}{2\pi}$ full turns $= \frac{600}{2(3.142)} = 95$.
   $v = 10(0.4) = 4$ m/s.

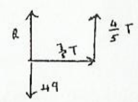
Exercise 10.b

1. $T = \frac{mv^2}{r} = \frac{8(25)}{2} = 100$ N.
2. (i) $\omega = 2\pi$ radians per second $= \frac{44}{7}$ rads/sec.
   (ii) $T = m\omega^2 r = 7(\frac{44}{7})^2 (\frac{7}{8}) = 242$ N.
3.

Forces



Resolved



(i) $|pq|^2 = |op|^2 + |oq|^2 \Rightarrow 25 = 16 + |oq|^2 \Rightarrow |oq| = 3$m

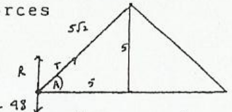
(ii) 1. $R + \frac{4}{5}T = 49$

2. $F_c = \frac{mv^2}{r} \Rightarrow \frac{3}{5}T = \frac{5(3)^2}{3} \Rightarrow T = 25$ N

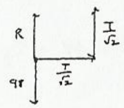
1. $\Rightarrow R + \frac{4}{5}(25) = 49 \Rightarrow R = 29$ N

4. contd ...

Forces



Resolved



(i) $\sin A = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow A = 45^\circ$.

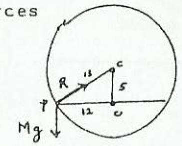
(ii) 1: $R + \frac{T}{\sqrt{2}} = 98$,

2: $Fc = m\omega^2 r \Rightarrow \frac{T}{\sqrt{2}} = (10)(1)(5) \Rightarrow T = 50\sqrt{2}$ N.

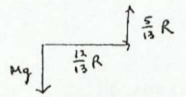
(iii) 1: $\Rightarrow R + 50 = 98 \Rightarrow R = 48$ N.

5.

Forces



Resolved



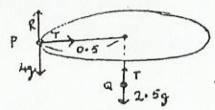
1: $Mg = \frac{5}{13}R \Rightarrow R = \frac{13Mg}{5}$.

2: $Fc = \frac{mv^2}{r} \Rightarrow \frac{12}{13}R = \frac{Mv^2}{12}$.

But $R = \frac{13Mg}{5}$.

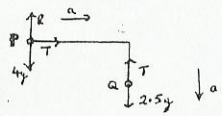
$\therefore \frac{12}{13}(\frac{13Mg}{5}) = \frac{Mv^2}{12} \Rightarrow v = \sqrt{\frac{144g}{5}} = 16.8$ m/s.

6.



Since Q is in equilibrium, $T = 2.5g = 24.5$ N.

For P, $Fc = m\omega^2 r \Rightarrow T = 4\omega^2(0.5) \Rightarrow 24.5 = 2\omega^2 \Rightarrow \omega = \sqrt{12.25} = 3.5$ rads/sec.



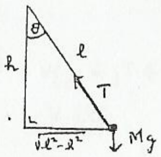
Q: $2.5g - T = (2.5)a$
P: $T = 4a$
Adding gives: $2.5g = 6.5a \Rightarrow a = \frac{5}{13}g = \frac{49}{13}$m/s².

$u = 0$, $a = \frac{49}{13}$, $s = 0.5$, $v = ?$

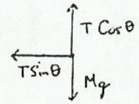
$v^2 = u^2 + 2as \Rightarrow v^2 = 0 + 2(\frac{49}{13})(0.5) \Rightarrow v = \frac{7}{\sqrt{13}}$

7. (i)

Forces



Resolved



7. contd ...

Since $\cos\theta = \frac{h}{l}$, $\sin\theta = \frac{\sqrt{l^2 - h^2}}{l}$

(ii) 1. $T\cos\theta = Mg \Rightarrow \frac{hT}{l} = Mg \Rightarrow T = \frac{Mgl}{h}$

2. $T\sin\theta = M\omega^2(\sqrt{l^2 - h^2}) \Rightarrow T\left(\frac{\sqrt{l^2-h^2}}{l}\right) = M\omega^2\sqrt{l^2 - h^2}$.

$\Rightarrow T = M\omega^2 l$.

(iii) Equating the two values of T gives:

$\frac{Mgl}{h} = M\omega^2 l \Rightarrow \frac{g}{h} = \omega^2 \Rightarrow h = \frac{g}{\omega^2}$.

(iv) $h \geqslant 0.25 \Rightarrow \frac{g}{\omega^2} \geqslant 0.25 \Rightarrow \omega^2 \leqslant \frac{g}{0.25} \Rightarrow \omega \leqslant 6.3$ rads/sec.

EXERCISE 10.C
=============

1. (i) $\frac{Gm_1m_2}{r^2} = \frac{m_1v^2}{r} \Rightarrow v = \sqrt{\frac{Gm_2}{r}}$.

$\Rightarrow v = \sqrt{\frac{6.67 \times 10^{-11} \times 2 \times 10^{30}}{1.5 \times 10^{11}}} = 29{,}830$ m/s.

(ii) TIME (in days) $= \frac{2\pi r}{v \times 86400} = \frac{2(3.142)(1.5 \times 10^{11})}{29830 \times 86400} = 365.7$ days

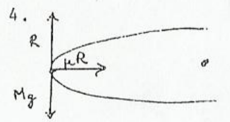
2. The time for one orbit = 1 day = 86400 seconds

$v = \sqrt{\frac{Gm}{r}} \Rightarrow$ Time, $T = \frac{2\pi r}{\sqrt{\frac{Gm}{r}}} \Rightarrow T^2 = \frac{4\pi^2 r^3}{Gm} \Rightarrow (86400)^2 =$

$\frac{4\pi r^3}{6.67 \times 10^{-11} \times 6 \times 10^{24}} \Rightarrow r^3 = \frac{(86400)^2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{4\pi^2}$

$\Rightarrow r = 42{,}300{,}000$m = 42300km. Height = 42,300 − 6,370 = 35,930 km.

3. As in Q2.

4. 

1: $R = Mg$

2: $\mu R = \frac{Mv^2}{r} \Rightarrow \mu Mg = \frac{M(21)^2}{50} \Rightarrow \mu = 0.9$

5. (a) 1: $R = Mg$.

2: $\mu R = \frac{Mv^2}{r} \Rightarrow \frac{2}{3}(Mg) = \frac{Mv^2}{24} \Rightarrow v^2 = 16g$

$\Rightarrow v = 4\sqrt{g} = 12.52$ m/s.

(b) 1: $R = Mg$.

2: $\mu R = \frac{Mv^2}{r} \Rightarrow \mu(Mg) = \frac{M(7)^2}{24} \Rightarrow \mu = \frac{5}{24}$.

6. 1. $R = Mg$.

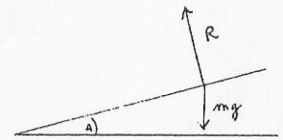
2. $\mu R = M\omega^2 r \Rightarrow \frac{1}{8}(Mg) = M\omega^2(0.1) \Rightarrow \omega^2 = 12.25 \Rightarrow \omega = 3.5$ rads/sec.

7. 1: $R = mg$

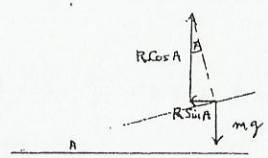
2: $\mu R = \frac{mv^2}{r} \Rightarrow \frac{1}{2}(mg) = \frac{m(36)}{r} \Rightarrow r = \frac{72}{g} = \frac{720}{98} = \frac{360}{49} = 7.35$m

8. $\tan A = \frac{v^2}{gr} = \frac{49}{(9.8)5} = 1 \Rightarrow A = 45°$

9. Forces



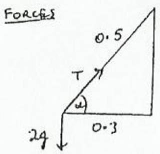
Resolved



1. $R\cos A = mg$.
2. $R\sin A = \frac{mv^2}{r}$. Dividing 2 by 1 gives $\tan A = \frac{v^2}{gr}$.

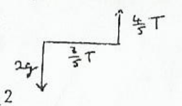
10. $\tan A = \frac{v^2}{gr} \Rightarrow \frac{1}{8} = \frac{4900}{(9.8)r} \Rightarrow r = 4000$m = 4km.

11. (a) Forces



$\cos A = \frac{0.3}{0.5} = \frac{3}{5} \Rightarrow \sin A = \frac{4}{5}$

Resolved



(i) Acceleration $= \frac{v^2}{r} = \frac{v^2}{0.3} = \frac{10v^2}{3}$

(ii) Centripetal force $= \frac{mv^2}{r} = \frac{2v^2}{0.3} = \frac{20v^2}{3}$

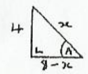
(iii) (i) $\frac{3}{5}T$ (ii) $\frac{4}{5}T$

1: $\frac{4}{5}T = 2g \Rightarrow T = 2.5g = 2.5(9.8) = 24.5$

2: $\frac{3}{5}T = \frac{20v^2}{3} \Rightarrow \frac{3}{5}(24.5) = \frac{20v^2}{3} \Rightarrow v = \sqrt{2.205} = 1.48$ m/s

(b) $\tan A = \frac{v^2}{gr} \Rightarrow \frac{5}{12} = \frac{v^2}{10(4800)} \Rightarrow v = \sqrt{20{,}000}$ m/s

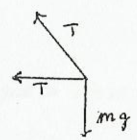
12. Let $|pr| = x \therefore |qr| = 8 - x$



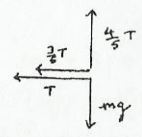
By Pythagoras, $x^2 = (4)^2 + (8-x)^2 \Rightarrow x^2 = 16 + 64 - 16x + x^2 \Rightarrow x = 5$.

$\therefore \sin A = \frac{4}{5}$, $\cos A = \frac{3}{5}$

Forces



Resolved



12. contd ...

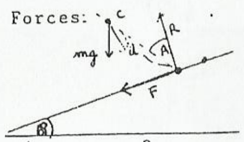
1: $\frac{4}{5}T = mg \Rightarrow T = \frac{5mg}{4}$

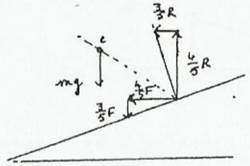
2: $Fc = m\omega^2 r \Rightarrow (T + \frac{3}{5}T) = m\omega^2(3) \Rightarrow \frac{8}{5}T = 3m\omega^2$. But $T = \frac{5mg}{4}$

$\Rightarrow \frac{8}{5}(\frac{5mg}{4}) = 3m\omega^2 \Rightarrow \omega^2 = \frac{2g}{3} \Rightarrow \omega = \sqrt{\frac{2g}{3}}$ rads/sec

13.

Since Tan B = $\frac{3}{4}$, Cos B = $\frac{4}{5}$, Sin B = $\frac{3}{5}$

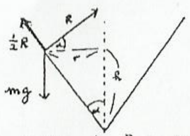
Forces: 

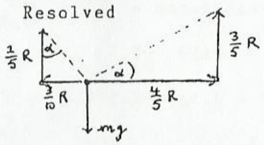
Resolved: 

1. $\frac{4}{5}R = mg + \frac{3}{5}F \Rightarrow 4R = 5mg + 3F \Rightarrow 4R - 3F = 5mg = 5(50)(9.8) = 2450.$
2. $Fc = \frac{mv^2}{r} \Rightarrow (\frac{3}{5}R + \frac{4}{5}F) = \frac{(50)(14)^2}{20} \Rightarrow 3R + 4F = 2450.$
3. Taking moments about c (using unresolved forces) $R(dSinA) = F(dCosA) \Rightarrow TanA = \frac{F}{R}$.

Solving 1 and 2 gives: R = 686 N, F = 98 N.
$Tan\ A = \frac{F}{R} = \frac{98}{686} = \frac{1}{7}$,

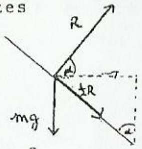
14. (i) Since Tan R = $\frac{3}{4}$, Sin = $\frac{3}{5}$, Cos = $\frac{4}{5}$.

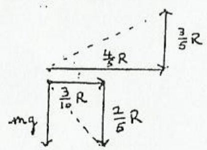
Forces 

Resolved 

1. $\frac{3}{5}R + \frac{2}{5}R = mg \Rightarrow R = mg$
2. $Fc = m\omega^2 r \Rightarrow (\frac{4}{5}R - \frac{3}{10}R) = m(7)^2 r \Rightarrow \frac{1}{2}R = 49mr$ But R = mg
$\Rightarrow \frac{1}{2}mg = 49mr \Rightarrow r = \frac{g}{98} = \frac{1}{10}$.

But $Tan\alpha = \frac{3}{4} = \frac{r}{h} \Rightarrow h = \frac{4}{3}r = \frac{4}{3}(\frac{1}{10}) = \frac{2}{15}m$,

(ii) Forces 

Resolved 

1: $\frac{3}{5}R = \frac{2}{5}R + mg \Rightarrow R = 5mg$

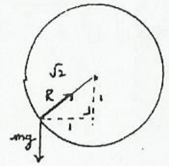
2: $Fc = m\omega^2 r \Rightarrow (\frac{4}{5}R + \frac{3}{10}R) = m(49)(r) \Rightarrow \frac{11}{10}R = 49mr$. But r = 5mg

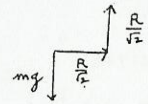
$\therefore \frac{11}{10}(5mg) = 49mr \Rightarrow r = \frac{11g}{98} = \frac{11}{10}$,

14. contd ...

$h = \frac{4}{3}r = \frac{4}{3}(\frac{11}{10}) = \frac{22}{15}m.$

15.

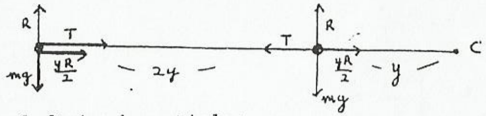
(a) Forces 

Resolved, 

1: $\frac{R}{\sqrt{2}} = mg$,

2: $Fc = m\omega^2 r = \frac{R}{\sqrt{2}} = m\omega^2(1) \therefore mg = m\omega^2 \Rightarrow \omega^2 = g \Rightarrow \omega = \sqrt{g}$ rads/sec.

(b)

(i) 

(ii) The left hand particle:

1: R = mg 2. $Fc = m\omega^2 r \Rightarrow T + \frac{yR}{2} = m\omega^2(3y)$

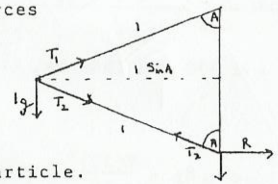
$\Rightarrow T + \frac{ymg}{2} = 3m\omega^2 y$ ..... Equation A

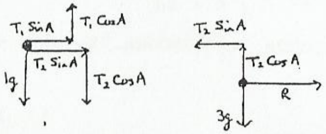
The right - hand particle:

1: R = mg.

2: $Fc = m\omega^2 r \Rightarrow \frac{yR}{2} - T = m\omega^2 y \Rightarrow \frac{ymg}{2} - T = m\omega^2 y$, ... Equation B

Adding equation A and B gives $ymg = 4m\omega^2 y \Rightarrow \omega^2 = \frac{g}{4} \Rightarrow \omega = \sqrt{g}/2$

16. Forces 

Resolved 

The 1kg particle.

1. Ups = Downs $\Rightarrow T_1 CosA = T_2 CosA + g$.
2. $Fc = m\omega^2 r \Rightarrow T_1 SinA + T_2 SinA = (1)(14)^2(1SinA) \Rightarrow T_1 + T_2 = 196$.

The 3kg particle:

3. Ups = Downs $\Rightarrow T_2 CosA = 3g$
4. Lefts = Rights $\Rightarrow T_2 SinA = R$

3. $\Rightarrow g = \frac{1}{3}T_2 CosA$. Putting this into equation 1 gives:
$T_1 CosA = T_2 CosA + \frac{1}{3}T_2 CosA \Rightarrow T_1 = \frac{4}{3}T_2 \therefore$ Equation 2 reads:

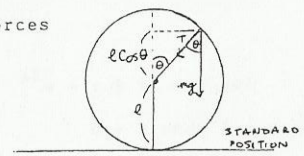
$\frac{4}{3}T_2 + T_2 = 196 \Rightarrow T_2 = 84\ N \Rightarrow T_1 = 112\ N$

Equation 3 reads: $T_2 CosA = 3g \Rightarrow 84CosA = 3(9.8) \Rightarrow CosA = \frac{29.4}{84} = \frac{7}{20} = 0.35$

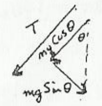
$\therefore A = 70^\circ$.

EXERCISE 10.D

1. Forces



Resolved



Let θ = the angle with the upward vertical.

1. $T + mg\cos\theta = \frac{mv^2}{l}$.

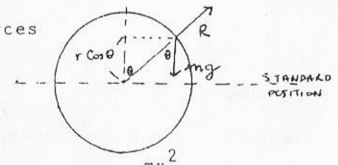
2. $Mg(0) + \frac{1}{2}m(3gl) = mg(l + l\cos\theta) + \frac{1}{2}mv^2 \Rightarrow mv^2 = mgl - 2mgl\cos\theta$.

Putting this result into equation 1 gives:

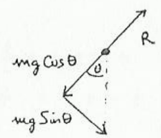
$T + mg\cos\theta = mg - 2mg\cos\theta \Rightarrow T = mg - 3mg\cos\theta$.

When the string goes slack, $T = 0 \Rightarrow 0 = mg - 3mg\cos\theta \Rightarrow \cos\theta = \frac{1}{3}$.

2. Forces



Resolved



1. $mg\cos\theta - R = \frac{mv^2}{r}$.

2. $mgr + \frac{1}{2}m(\frac{gr}{2}) = mgr\cos\theta + \frac{1}{2}mv^2 \Rightarrow mv^2 = \frac{5}{2}mgr - 2mgr\cos\theta$.

Putting this result into equation 1 gives:

$mg\cos\theta - R = \frac{5}{2}mg - 2mg\cos\theta$.

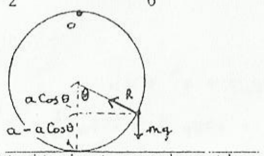
when the particle leaves the sphere, $R = 0$ $\therefore mg\cos\theta = \frac{5}{2}mg - 2mg\cos\theta$

$\Rightarrow \cos\theta = \frac{5}{6} \Rightarrow \theta = 33^\circ\ 34'$.

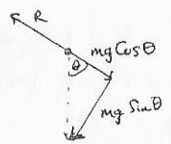
Putting this result back into equation 2 gives:

$mv^2 = \frac{5}{2}mgr - 2mgr(\frac{5}{6}) \Rightarrow v = \sqrt{\frac{5gr}{6}}$.

3. Forces:



Resolved:



Assume that it just reaches the top.

$Mg0 + \frac{1}{2}mu^2 = mg(2a) + \frac{1}{2}m(0)^2 \Rightarrow u^2 = 4ga \Rightarrow u = \sqrt{4ga}$.

1: $R - mg\cos\theta = \frac{mv}{a}$.

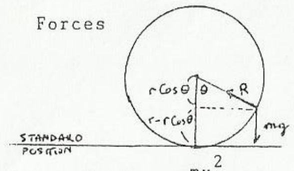
2: $mg(0) + \frac{1}{2}m(4ga) = mg(a - a\cos\theta) + \frac{1}{2}mv^2 \Rightarrow mv^2 = 2mga + 2ma\cos\theta$.

When $\theta = 60^\circ$, $\cos\theta = \frac{1}{2} \Rightarrow mv^2 = 3mga$.

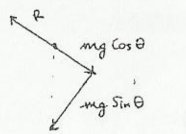
Putting these results into equation 1 gives

$R - mg(\frac{1}{2}) = \frac{3mga}{a} \Rightarrow R = \frac{7mg}{2}$.

4. Forces



Resolved



1: $R - mg\cos\theta = \frac{mv^2}{r}$.

2: $mg(0) + \frac{1}{2}m(\frac{7gr}{2}) = mg(r - r\cos\theta) + \frac{1}{2}mv^2 \Rightarrow mv^2 = \frac{3}{2}mgr + 2mgr\cos\theta$.

Putting this result into equation 1 gives

$R - mg\cos\theta = \frac{3}{2}mg + 2mg\cos\theta$.

When the marble leaves the sphere, $R = 0$.

$\therefore -mg\cos\theta = \frac{3}{2}mg + 2mg\cos\theta \Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = 120^\circ$

It has risen $r - r\cos\theta = r - r(-\frac{1}{2}) = \frac{3}{2}r$.

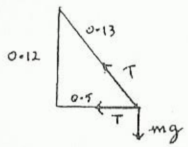
5. (a) Let $|pr| = x$ $\therefore |rq| = 0.18 - x$

$|pr|^2 = |pq|^2 + |qr|^2 \Rightarrow x^2 = (0.12)^2 + (0.18 - x)^2$

$\Rightarrow x^2 = 0.0144 + 0.0324 - 0.36x + x^2 \Rightarrow x = 0.13$.

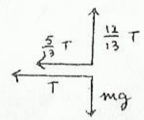
$\therefore |pr| = 0.13$, $|rq| = 0.05$.

Forces



$\sin A = \frac{12}{13}$, $\cos A = \frac{5}{13}$

Forces



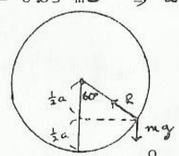
1. $\frac{12}{13}T = mg \Rightarrow T = \frac{13mg}{12}$.

2: $Fc = m\omega^2 r \Rightarrow (\frac{5}{13}T + T) = m\omega^2(0.05) \Rightarrow \frac{18}{13}T = 0.05\ m\omega^2$

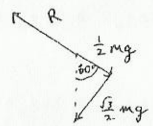
But $T = \frac{13mg}{12}$, therefore

$\frac{18}{13}(\frac{13mg}{12}) = 0.05\ m\omega^2 \Rightarrow \omega = \sqrt{30g} = \sqrt{294}$ rads/sec.

(b) Forces:



Resolved:



Let v = the speed when $\theta = 60^\circ$

1. $R - \frac{1}{2}mg = \frac{mv^2}{a}$

2. $Mg(0) + \frac{1}{2}mu^2 = mg(\frac{1}{2}a) + \frac{1}{2}mu^2 \Rightarrow mv^2 = mu^2 - mga$

Putting this result into equation 1 gives:

$R - \frac{1}{2}mg = \frac{mu^2 - mga}{a} \Rightarrow R = \frac{1}{2}mg + \frac{mu^2}{a} - mg = \frac{mu^2}{a} - \frac{1}{2}mg = m(\frac{u^2}{a} - \frac{g}{2})$

CHAPTER 11. DIFFERENTIAL EQUATIONS.

Exercise 11.A

1. (i) $\log_e 4 + \log_e 3 = \log_e 12$.

(ii) $\log_e 6 - \log_e 7 = \log_e \frac{6}{7}$.

(iii) $2\log_e 3 + 3\log_e 2 = \log_e 3^2 + \log_e 2^3 = \log_e 9 + \log_e 8 = \log_e 72$.

(iv) $5\log_e 2 - 2\log_e 5 = \log_e 2^5 - \log_e 5^2 = \log_e 32 - \log_e 25 = \log_e \frac{32}{25}$.

(v) $\frac{1}{2}\log_e 4 + \frac{1}{3}\log_e 27 = \log_e 4^{\frac{1}{2}} + \log_e 27^{\frac{1}{3}} = \log_e 2 + \log_e 3 = \log_e 6$.

(vi) $\frac{1}{2}\log_e 64 - \frac{1}{3}\log_e 64 = \log_e 8 - \log_e 4 = \log_e 2$.

(vii) $2\log_e 10 + \log_e 6 - 3\log_e 4 = \log_e 100 + \log_e 6 - \log_e 64 = \log_e\left(\frac{100 \times 6}{64}\right) = \log_e\left(\frac{75}{8}\right)$.

(viii) $\log_e x^2 + \log_e x = \log_e x^3$.

(ix) $\frac{1}{2}\log_e x - \log_e 7 + \log_e 2 = \log_e \sqrt{x} - \log_e 7 + \log_e 2 = \log_e \frac{2\sqrt{x}}{7}$.

2. (i) $e^x = e \Rightarrow x = 1$.

(ii) $x = e^2$.

(iii) $e^x = \frac{1}{e} \Rightarrow x = -1$.

(iv) $e^x = \sqrt[3]{e} \Rightarrow x = \frac{1}{3}$.

(v) $e^{\log_e x} = 8 \Rightarrow x = 8$ (since e and $\log_e$ are inverse functions).

(vi) $\log_e(e^4) = x \Rightarrow e^x = e^4 \Rightarrow x = 4$.

(vii) $e^{\log_e 2} = x \Rightarrow 2 = x$.

(viii) $e^{\frac{1}{2}\log_e x} = 7 \Rightarrow e^{\log_e \sqrt{x}} = 7 \Rightarrow \sqrt{x} = 7 \Rightarrow x = 49$.

(ix) $e^{\log_e x} = 9 \Rightarrow e^{\log_e x^2} = 9 \Rightarrow x^2 = 9 \Rightarrow x = +3$.
(-3 is not possible since $\log_e(-3)$ does'nt exist).

(x) $e^{3\log_e 4} = x \Rightarrow e^{\log_e 64} = x \Rightarrow x = 64$.

(xi) $\log_e x = 1 + \log_e 2 \Rightarrow \log_e x - \log_e 2 = 1 \Rightarrow \log_e \frac{x}{2} = 1 \Rightarrow$
$\frac{x}{2} = e^1 \Rightarrow x = 2e$.

(xii) $\log_e x = 3 - \log_e 5 \Rightarrow \log_e x + \log_e 5 = 3 \Rightarrow \log_e 5x = 3$
$\Rightarrow 5x = e^3 \Rightarrow x = \frac{1}{5}e^3$.

(xiii) $2\log_e x = 1 - \log_e 3 \Rightarrow \log_e x^2 = 1 - \log_e 3 \Rightarrow \log_e x^2 + \log_e 3 = 1$
$\Rightarrow \log_e 3x^2 = 1 \Rightarrow 3x^2 = e \Rightarrow x = \sqrt{\frac{e}{3}}$.

(xiv) $\frac{1}{2}\log_e x + 1 = \frac{3}{2}\log_e 3 \Rightarrow \log_e x + 2 = 3\log_e 3 \Rightarrow \log_e x - 3\log_e 3 = -2$
$\Rightarrow \log_e\left(\frac{x}{27}\right) = -2 \Rightarrow \frac{x}{27} = e^{-2} \Rightarrow x = 27e^{-2}$.

EXERCISE 11.B

1. $\frac{dy}{dx} = 3y \Rightarrow \int\frac{dy}{y} = \int 3dx \Rightarrow \log_e y = 3x + c$.
But $y = 1$ when $x = 0 \Rightarrow \log_e 1 = 0 + c \Rightarrow c = 0 \therefore \log_e y = 3x \Rightarrow y = e^{3x}$.

2. $\frac{dy}{dx} = 5y \Rightarrow \int\frac{dy}{y} = \int 5dx \Rightarrow \log_e y = 5x + c$. But $y = 2$ when $x = 0$
$\Rightarrow \log_e 2 = c$.
$\therefore \log_e y = 5x + \log_e 2 \Rightarrow \log_e y - \log_e 2 = 5x \Rightarrow \log_e \frac{y}{2} = 5x \Rightarrow \frac{y}{2} = e^{5x}$
$\Rightarrow y = 2e^{5x}$.

3. $\frac{dy}{dx} = 2x\sqrt{1 - y^2} \Rightarrow \int\frac{dy}{\sqrt{1 - y^2}} = \int 2x\,dx \Rightarrow \sin^{-1}y = x^2 + c$.
But $y = 1$ when $x = 0 \Rightarrow \sin^{-1}1 = c \Rightarrow c = \frac{\pi}{2} \therefore \sin^{-1}y = x^2 + \frac{\pi}{2} \Rightarrow$
$y = \sin\left(x^2 + \frac{\pi}{2}\right)$.

4. $\frac{dx}{dt} = tx \Rightarrow \int\frac{dx}{x} = \int t\,dt \Rightarrow \log_e x = \frac{t^2}{2} + c$. But $x = \sqrt{e}$ when $t = 1$
$\Rightarrow \log_e \sqrt{e} = 1 + c \Rightarrow \frac{1}{2} = \frac{1}{2} + c \Rightarrow c = 0. \therefore \log_e x = \frac{1}{2}t^2 \Rightarrow x = e^{\frac{1}{2}t^2}$.

5. $\frac{dy}{dx} = \frac{y}{x} \Rightarrow \int\frac{dy}{y} = \int\frac{dx}{x} \Rightarrow \log_e y = \log_e x + c$. But $y = 3$ when $x = 1$
$\Rightarrow \log_e 3 = \log_e 1 + c \Rightarrow c = \log_e 3. \therefore \log_e y = \log_e x + \log_e 3 \Rightarrow$
$\log_e y = \log_e 3x \Rightarrow y = 3x$.

6. $\frac{ds}{dt} + \frac{\sin t}{s} = 0 \Rightarrow \frac{ds}{dt} = \frac{-\sin t}{s} \Rightarrow \int s\,ds = \int -\sin t\,dt \Rightarrow \frac{s^2}{2} = \cos t + c$
But $s = \sqrt{2}$ when $t = \frac{\pi}{3} \Rightarrow \frac{2}{2} = \frac{1}{2} + c \Rightarrow c = \frac{1}{2}. \therefore \frac{s^2}{2} = \cos t + \frac{1}{2} \Rightarrow$
$s^2 = 2\cos t + 1 \Rightarrow s = \sqrt{2\cos t + 1}$.

7. $\frac{dy}{dx} - 4x^3y = 0 \Rightarrow \frac{dy}{dx} = 4x^3y \Rightarrow \int\frac{dy}{y} = \int 4x^3dx \Rightarrow \log_e y = x^4 + c$.
But $y = 3$ when $x = 0 \Rightarrow \log_e 3 = c \therefore \log_e y = x^4 + \log_e 3 \Rightarrow \log_e \frac{y}{3} = x^4$
$\Rightarrow \frac{y}{3} = e^{x^4} \Rightarrow y = 3e^{x^4}$

8. $\frac{y}{x^2}\frac{dy}{dx} = 1 \Rightarrow \int y\,dy = \int x^2dx \Rightarrow \frac{y^2}{2} = \frac{x^3}{3} + c$. But $x = 1$ when $y = 0$
$\Rightarrow 0 = \frac{1}{3} + c \Rightarrow c = -\frac{1}{3} \therefore \frac{y^2}{2} = \frac{x^3}{3} - \frac{1}{3} \Rightarrow y^2 = \frac{2}{3}(x^3 - 1) \Rightarrow y = \sqrt{\frac{2}{3}(x^3 - 1)}$.

9. $\frac{dy}{dx} - \frac{\text{Cos}x}{y} = 0 \Rightarrow \frac{dy}{dx} = \frac{\text{Cos}x}{y} \Rightarrow \int y\,dy = \int \text{Cos}x\,dx \Rightarrow \frac{y^2}{2} = \text{Sin}\,x + c.$

But $y = 1$ when $x = \frac{\pi}{2} \Rightarrow \frac{1}{2} = 1 + c \Rightarrow c = -\frac{1}{2}. \therefore \frac{y^2}{2} = \text{Sin}x - \frac{1}{2} \Rightarrow$

$y^2 = 2\text{Sin}x - 1 \Rightarrow y = \sqrt{2\text{Sin}x - 1}.$

10. $\frac{dy}{dx} - \frac{y+1}{x} = 0 \Rightarrow \frac{dy}{dx} = \frac{y+1}{x} \Rightarrow \int \frac{dy}{y+1} = \int \frac{dx}{x} \Rightarrow \log_e(y+1) = \log_e x + c.$

But $x = 4$ when $y = 0 \Rightarrow \log_e 1 = \log_e 4 + c \Rightarrow c = -\log_e 4$

$\therefore \log_e(y+1) = \log_e x - \log_e 4 \Rightarrow \log_e(y+1) = \log_e \frac{x}{4} \Rightarrow y+1 = \frac{x}{4} \Rightarrow y = \frac{x}{4} - 1.$

11. $\frac{dy}{dx} - y^2\text{Sin}x = 0 \Rightarrow \frac{dy}{dx} = y^2\text{Sin}x \Rightarrow \int \frac{dy}{y^2} = \int \text{Sin}\,x\,dx \Rightarrow -\frac{1}{y} = -\text{Cos}x + c$

$\Rightarrow \frac{1}{y} = \text{Cos}x - c$. But $x = \pi$ when $y = 1 \Rightarrow 1 = -1 - c \Rightarrow c = -2$

$\therefore \frac{1}{y} = \text{Cos}x + 2 \Rightarrow y = \frac{1}{\text{Cos}x + 2}.$

12. $xy\frac{dy}{dx} = (5 + y^2) \Rightarrow \int \frac{y}{5+y^2}\,dy = \int \frac{dx}{x} \Rightarrow \frac{1}{2}\log_e(5+y^2) = \log_e x + c$

But $y = 2$ when $x = 1 \Rightarrow \frac{1}{2}\log_e 9 = \log_e 1 + c \Rightarrow c = \frac{1}{2}\log_e 9 = \log_e\sqrt{9} =$
$\log_e 3. \therefore \frac{1}{2}\log_e(5+y^2) = \log_e x + \log_e 3 \Rightarrow \log_e \sqrt{5 + y^2} = \log_e 3x$

$\Rightarrow \sqrt{5 + y^2} = 3x \Rightarrow 5 + y^2 = 9x^2 \Rightarrow y^2 = 9x^2 - 5 \Rightarrow y = \sqrt{9x^2 - 5}.$

13. $v\frac{dv}{dx} = \text{Cos}^2 x \Rightarrow \int v\,dv = \int \text{Cos}^2 x\,dx \Rightarrow \frac{v^2}{2} = \frac{1}{2}(x + \frac{1}{2}\text{Sin}2x) + c \Rightarrow$

$v^2 = x + \frac{1}{2}\text{Sin}2x + 2c.$ But $v = 1$ when $x = 0 \Rightarrow 1 = 0 + 0 + 2c \Rightarrow c = \frac{1}{2}$

$\therefore v^2 = x + \frac{1}{2}\text{Sin}2x + 1 \Rightarrow v = \sqrt{x + \frac{1}{2}\text{Sin}2x + 1}$

14. $\frac{dy}{dx} = y\text{Sin}x \Rightarrow \int \frac{dy}{y} = \int \text{Sin}x\,dx \Rightarrow \log_e y = -\text{Cos}x + c$. But $y = \sqrt{e}$ when
$x = \frac{\pi}{3} \Rightarrow \log_e \sqrt{e} = -\frac{1}{2} + c \Rightarrow \frac{1}{2} = -\frac{1}{2} + c \Rightarrow c = 1 \;\therefore \log_e y =$
$-\text{Cos}x + 1 \Rightarrow y = e^{1-\text{Cos}x}$

EXERCISE 11.C

1. $\frac{d^2y}{dx^2} = 2\frac{dy}{dx} \Rightarrow \frac{dv}{dx} = 2v,$ where $v = \frac{dy}{dx} \quad \therefore \int \frac{dv}{v} = \int 2\,dx \Rightarrow$

$\log_e v = 2x + c.$ But $v = 1$ when $x = 0 \Rightarrow 0 = 0 + c \Rightarrow c = 0$

$\therefore \log_e v = 2x \Rightarrow \boxed{v = e^{2x}}$ End of step 1.

$v = e^{2x} \Rightarrow \frac{dy}{dx} = e^{2x} \Rightarrow \int dy = \int e^{2x}\,dx \Rightarrow y = \frac{1}{2}e^{2x} + k,$

But $y = 0$ when $x = 0 \Rightarrow 0 = \frac{1}{2}e^0 + k \Rightarrow k = -\frac{1}{2} \;\therefore y = \frac{1}{2}e^{2x} - \frac{1}{2} = \frac{1}{2}(e^{2x} - 1)$

2. $\frac{d^2x}{dt^2} = (\frac{dx}{dt})^2 \Rightarrow \frac{dv}{dt} = v^2$, where $v = \frac{dx}{dt} \quad \therefore \int \frac{dv}{v^2} = \int dt$

$\Rightarrow -\frac{1}{v} = t + c \quad$ But $v = 1$ when $t = 0 \Rightarrow -\frac{1}{1} = 0 + c \Rightarrow c = -1$

$\therefore -\frac{1}{v} = t - 1 \Rightarrow \frac{1}{v} = 1 - t \Rightarrow \boxed{v = \frac{1}{1 - t}}$

End of Step 1.

$v = \frac{1}{1 - t} \Rightarrow \frac{dx}{dt} = \frac{1}{1 - t} \Rightarrow \int dx = \int \frac{dt}{1 - t} \Rightarrow x = -\log_e(1-t) + c$

But $x = 0$ when $t = 0 \Rightarrow 0 = -\log_e(1) + c \Rightarrow c = 0$

$\therefore x = -\log_e(1-t) = \log_e(1-t)^{-1} = \log_e(\frac{1}{1 - t}).$

3. $\frac{d^2s}{dt^2} = 6 \Rightarrow \frac{dv}{dt} = 6$ where $v = \frac{ds}{dt} \Rightarrow \int dv = \int 6\,dt \Rightarrow v = 6t + c$

But $v = 4$ when $t = 0 \Rightarrow 4 = 0 + c \Rightarrow c = 4 \;\therefore \boxed{v = 6t + 4}$

End of Step 1

$v = 6t + 4 \Rightarrow \frac{ds}{dt} = 6t + 4 \Rightarrow ds = (6t + 4)dt \Rightarrow s = 3t^2 + 4t + k$

But $s = 0$ when $t = 0 \Rightarrow k = 0 \;\therefore s = 3t^2 + 4t$

4. $\frac{d^2s}{dt^2} = a \Rightarrow \frac{dv}{dt} = a,$ where $v = \frac{ds}{dt} \;\therefore \int dv = \int a\,dt \Rightarrow v = at + c$

But $v = u$ when $t = 0 \Rightarrow u = 0 + c \Rightarrow c = u \;\therefore \boxed{v = at + u.}$

End of Step 1

$v = at + u \Rightarrow \frac{ds}{dt} = at + u \Rightarrow \int ds = \int (at + u)dt \Rightarrow s = \frac{1}{2}at^2 + ut + k$

But $s = 0$ when $t = 0 \Rightarrow k = 0 \;\therefore s = ut + \frac{1}{2}at^2$. Now, where did I see that before?

5. $\frac{d^2s}{dt^2} = -(\frac{ds}{dt})^2 \Rightarrow \frac{dv}{dt} = -v^2,$ where $v = \frac{ds}{dt} \;\therefore \int \frac{dv}{v^2} = \int -dt$

$\Rightarrow -\frac{1}{v} = -t + c.$ But $v = \frac{1}{2}$ when $t = 0 \Rightarrow -2 = 0 + c \Rightarrow c = -2$

$\therefore -\frac{1}{v} = -t - 2 \Rightarrow \frac{1}{v} = t + 2 \Rightarrow \boxed{v = \frac{1}{t + 2}}$ End of Step 1

$v = \frac{1}{t + 2} \Rightarrow \frac{ds}{dt} = \frac{1}{t + 2} \Rightarrow \int ds = \int \frac{dt}{t+2} \Rightarrow s = \log_e(t + 2) + k$

But $s = 0$ when $t = 0 \Rightarrow 0 = \log_e 2 + k \Rightarrow k = -\log_e 2$

$\therefore s = \log_e(t + 2) - \log_e 2 = \log_e(\frac{t+2}{2})$

6. $\frac{d^2s}{dt^2} = 1/\frac{ds}{dt} \Rightarrow \frac{dv}{dt} = \frac{1}{v}$ where $v = \frac{ds}{dt}$ $\therefore \int v\,dv = \int dt \Rightarrow \frac{1}{2}v^2 = t + c.$

But $v = 4$ when $t = 0 \Rightarrow c = 8$, $\therefore \frac{1}{2}v^2 = t + 8 \Rightarrow v^2 = 2t + 16 =$

$\Rightarrow \boxed{v = \sqrt{2t+16}} \Rightarrow \frac{ds}{dt} = \sqrt{2t+16}$

$\Rightarrow \int ds = \int \sqrt{2t+16}\,dt$

Using the substitution $u = 2t + 16 \Rightarrow \frac{1}{2}du = dt$, the solution is

$s = \frac{1}{3}(2t+16)^{3/2} + k$. But $s = 0$ when $t = 0 \Rightarrow 0 = \frac{1}{3}(16)^{3/2} + k$

$\Rightarrow k = \frac{-64}{3}$, $\therefore s = \frac{1}{3}(2t+16)^{3/2} - \frac{64}{3} = \frac{1}{3}((2t+16)^{3/2} - 64)$.

7. $\frac{d^2x}{dt^2} = (\frac{dx}{dt})^2 + 1 \Rightarrow \frac{dv}{dt} = v^2 + 1$, where $v = \frac{dx}{dt}$. $\therefore \int \frac{dv}{v^2+1} =$

$\int dt \Rightarrow \tan^{-1}v = t + c$. But $v = 0$ when $t = 0 \Rightarrow 0 = 0 + c \Rightarrow c = 0$

$\therefore \tan^{-1} v = t \Rightarrow \boxed{v = \tan t}$

$v = \operatorname{Tan} t \Rightarrow \frac{dx}{dt} = \tan t \Rightarrow \int dx = \int \tan t\,dt \Rightarrow x = -\log_e|\operatorname{Cos} t| + k$

But $x = 0$ when $t = 0 \Rightarrow 0 = -\log_e(1) + k \Rightarrow k = 0$.

$\therefore x = -\log_e|\operatorname{Cos} t|$ or $\log_e|\operatorname{Sec} t|$.

EXERCISE 11. D

1. $\frac{d^2y}{dx^2} = y \Rightarrow v\frac{dv}{dy} = y$, where $v = \frac{dy}{dx}$ $\therefore \int v\,dv = \int y\,dy \Rightarrow \frac{1}{2}v^2 = \frac{1}{2}y^2 + c$.

But $y = 1$, when $v = 1 \Rightarrow \frac{1}{2} = \frac{1}{2} + c \Rightarrow c = 0$.

$\therefore \frac{1}{2}v^2 = \frac{1}{2}y^2 \Rightarrow \boxed{v = y}$ ($v = -y$ won't work)

$v = y \Rightarrow \frac{dy}{dx} = y \Rightarrow \int \frac{dy}{y} = \int dx \Rightarrow \log_e y = x + k$

But $y = 1$ when $x = 0 \Rightarrow \log_e 1 = 0 + k \Rightarrow k = 0$ $\therefore \log_e y = x \Rightarrow y = e^x$.

2. $\frac{d^2s}{dt^2} = 9s \Rightarrow v\frac{dv}{ds} = 9s$, where $v = \frac{ds}{dt}$ $\therefore \int v\,dv = \int 9s\,ds \Rightarrow \frac{1}{2}v^2 = \frac{9}{2}s^2 + c$.

But $v = -6$ when $s = 2 \Rightarrow 18 = 18 + c \Rightarrow c = 0$.

$\therefore \frac{1}{2}v^2 = \frac{9}{2}s^2 \Rightarrow \boxed{v = -3s}$ ($v = 3s$ won't work)

$v = -3s \Rightarrow \frac{ds}{dt} = -3s \Rightarrow \int \frac{ds}{s} = \int -3dt \Rightarrow \log_e s = -3t + k$.

But $s = e$ when $t = 0 \Rightarrow \log_e e = 0 + k \Rightarrow k = 1$.

$\therefore \log_e s = -3t + 1 \Rightarrow s = e^{1-3t}$.

3. $\frac{d^2y}{dx^2} = -\frac{2}{y^5} \Rightarrow v\frac{dv}{dy} = -\frac{2}{y^5}$, where $v = \frac{dy}{dx}$. $\therefore \int v\,dv =$

$\int -2y^{-5}\,dy \Rightarrow \frac{1}{2}v^2 \Rightarrow \frac{1}{2}y^{-4} + c$.

But $v = 1$ when $y = 1 \Rightarrow \frac{1}{2} = \frac{1}{2} + c \Rightarrow c = 0$ $\therefore \frac{1}{2}v^2 = \frac{1}{2}y^{-4} \Rightarrow$

$\boxed{v = \frac{1}{y^2}}$ ($v = -\frac{1}{y^2}$ won't work).

$v = \frac{1}{y^2} \Rightarrow \frac{dy}{dx} = \frac{1}{y^2} \Rightarrow \int y^2\,dy = \int dx \Rightarrow \frac{1}{3}y^3 = x + k$.

But $y = 1$ when $x = \frac{1}{3} \Rightarrow \frac{1}{3} = \frac{1}{3} + k \Rightarrow k = 0$ $\therefore \frac{1}{3}y^3 = x \Rightarrow y^3 = 3x \Rightarrow y = \sqrt[3]{3x}$.

4. $\frac{d^2y}{dx^2} = -y \Rightarrow v\frac{dv}{dy} = -y$, where $v = \frac{dy}{dx}$ $\therefore \int v\,dv = \int -y\,dy \Rightarrow \frac{1}{2}v^2 =$

$-\frac{1}{2}y^2 + c$. But $v = 2$ when $y = 0 \Rightarrow 2 = 0 + c \Rightarrow c = 2$.

$\therefore \frac{1}{2}v^2 = -\frac{1}{2}y^2 + 2 \Rightarrow v^2 = -y^2 + 4 \Rightarrow \boxed{v = \sqrt{4 - y^2}}$

($v = -\sqrt{4 - y^2}$ won't work). $v = \sqrt{4 - y^2} \Rightarrow \frac{dy}{dx} = \sqrt{4 - y^2} \Rightarrow$

$\int \frac{dy}{\sqrt{4-y^2}} = \int dx \Rightarrow \operatorname{Sin}^{-1}\frac{y}{2} = x + k$. But $x = 0$ when $y = 0 \Rightarrow$

$0 = 0 + k \Rightarrow k = 0$ $\therefore \operatorname{Sin}^{-1}\frac{y}{2} = x \Rightarrow \frac{y}{2} = \operatorname{Sin} x \Rightarrow y = 2\operatorname{Sin} x$.

5. $\frac{d^2x}{dt^2} = 2x(9 + x^2) \Rightarrow v\frac{dv}{dx} = 2x(9 + x^2)$ when $v = \frac{dx}{dt}$

$\therefore \int v\,dv = \int 2x(9 + x^2)\,dx \Rightarrow \frac{1}{2}v^2 = \frac{1}{2}(9 + x^2)^2 + c$

(using the substitution : $u = 9 + x^2$). But $v = 9$ when $x = 0 \Rightarrow$

$\frac{81}{2} = \frac{81}{2} + c \Rightarrow c = 0$ $\therefore \frac{1}{2}v^2 = \frac{1}{2}(9 + x^2)^2 \Rightarrow \boxed{v = 9 + x^2}$ ($v = -(9+x^2)$ won't work).

$v = 9 + x^2 \Rightarrow \frac{dx}{dt} = 9 + x^2 \Rightarrow \int \frac{dx}{9+x^2} = \int dt \Rightarrow \frac{1}{3}\tan^{-1}\frac{x}{3} = t + k$

But $x = 3$ when $t = 0 \Rightarrow \frac{1}{3}\tan^{-1}(1) = 0 + k \Rightarrow k = \frac{\pi}{12}$.

$\therefore \frac{1}{3}\tan^{-1}\frac{x}{3} = t + \frac{\pi}{12} \Rightarrow \tan^{-1}(\frac{x}{3}) = 3t + \frac{\pi}{4} \Rightarrow x = 3\tan(3t + \frac{\pi}{4})$

6. $\frac{d^2s}{dt^2} = \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = \frac{dv}{ds}\,v = v\frac{dv}{ds}$. $v\frac{dv}{ds} = a \Rightarrow \int v\,dv = \int a\,ds$

$\Rightarrow \frac{1}{2}v^2 = as + c$ But $v = u$ when $s = 0 \Rightarrow \frac{1}{2}u^2 = 0 + c \Rightarrow c = \frac{1}{2}u^2$

$\therefore \frac{1}{2}v^2 = as + \frac{1}{2}u^2 \Rightarrow v^2 = 2as + u^2$.

7. $\frac{d^2x}{dt^2} = \frac{3x^2}{2} \Rightarrow v\frac{dv}{dx} = \frac{3x^2}{2}$ where $v = \frac{dx}{dt}$ $\therefore \int v\,dv = \int \frac{3}{2}x^2\,dx \Rightarrow$

$\frac{1}{2}v^2 = \frac{1}{2}x^3 + c$. But $v = -8$ when $x = 4 \Rightarrow 32 = 32 + c \Rightarrow c = 0$

$\therefore \frac{1}{2}v^2 = \frac{1}{2}x^3 \Rightarrow \boxed{v = -\sqrt{x^3}}$ ($v = \sqrt{x^3}$ won't work).

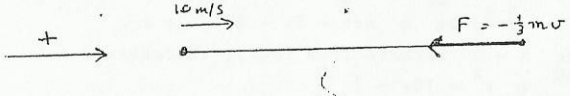
$v = -x^{3/2} \Rightarrow \frac{dx}{dt} = -x^{3/2} \Rightarrow \int x^{-3/2}\,dx = \int -dt \Rightarrow -2x^{-\frac{1}{2}} = -t + k$.

But $x = 4$ when $t = 0 \Rightarrow -1 = 0 + k \Rightarrow k = -1$ $\therefore -2x^{-\frac{1}{2}} = -t - 1$

$\Rightarrow 2x^{-\frac{1}{2}} = t + 1 \Rightarrow \frac{2}{\sqrt{x}} = t + 1 \Rightarrow \sqrt{x} = \frac{2}{t+1} \Rightarrow x = \left(\frac{2}{t+1}\right)^2$

EXERCISE 11.E

1. (i)



$F = ma \Rightarrow -\frac{1}{3}mv = ma \Rightarrow a = -\frac{1}{3}v \Rightarrow \frac{dv}{dt} = -\frac{1}{3}v \Rightarrow \int\frac{dv}{v} = \int -\frac{1}{3}dt$

$\Rightarrow \log_e v = -\frac{1}{3}t + c$. But $v = 10$ when $t = 0 \Rightarrow \log_e 10 = 0 + c \Rightarrow c = \log_e 10$.

$\therefore \log_e v = -\frac{1}{3}t + \log_e 10 \Rightarrow \log_e v - \log_e 10 = -\frac{1}{3}t \Rightarrow \frac{v}{10} = e^{-\frac{1}{3}t}$

$\Rightarrow v = 10e^{-\frac{1}{3}t}$ ..... equation 1 .

When $t = 3$, $v = 10e^{-1} = \frac{10}{e}$ m/s

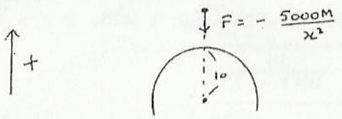
(ii) $\frac{ds}{dt} = 10e^{-\frac{1}{3}t}$ (from equation 1) $\Rightarrow \int ds = \int 10e^{-\frac{1}{3}t}dt \Rightarrow s = -30e^{-\frac{1}{3}t} + k$

But $s = 0$ when $t = 0 \Rightarrow 0 = -30(1) + k \Rightarrow k = 30$. $\therefore s = -30e^{-\frac{1}{3}t} + 30$

$= 30(1 - e^{-\frac{1}{3}t})$. When $t = 3$, $s = 30(1 - e^{-1}) = 30(1 - \frac{1}{e})$.

(iii) As $t \to \infty$, $e^{-\frac{1}{3}t} \longrightarrow 0$ $\therefore s \longrightarrow 30(1 - 0) = 30$m

2.



$F = ma \Rightarrow \frac{-5000M}{x^2} = Ma \Rightarrow a = \frac{-5000}{x^2}$ $\therefore v\frac{dv}{dx} = \frac{-5000}{x^2} \Rightarrow v\,dv =$

$-5000x^{-2}\,dx. \Rightarrow \frac{1}{2}v^2 = \frac{5000}{x} + c$. But $v = 20$ when $x = 10$ (the initial conditions)

$\therefore \frac{1}{2}(400) = \frac{5000}{10} + c \Rightarrow c = -300$ $\therefore \frac{1}{2}v^2 = \frac{5000}{x} - 300 \Rightarrow v^2 =$

$\frac{10000}{x} - 600$. When it is $\frac{30}{7}$ metres up, $x = 10 + \frac{30}{7} = \frac{100}{7}$.

$\therefore v^2 = \frac{10000}{100/7} - 600 = 100 \Rightarrow v = 10$ m/s.

As its greatest height, $v = 0$ $\therefore 0 = \frac{10000}{x} - 600 \Rightarrow x = \frac{10000}{600}$

$= \frac{50}{3}$m. $\therefore$ Height above the surface $= \frac{50}{3} - 10 = \frac{20}{3}$ m.

3. $v\frac{dv}{ds} = v^2 + 4 \Rightarrow \int\frac{v\,dv}{v^2+4} = \int ds \Rightarrow \frac{1}{2}\log_e(v^2 + 4) = s + c$.

But $v = 2$ when $s = 0 \Rightarrow \frac{1}{2}\log_e(8) = 0 + c \Rightarrow c = \frac{1}{2}\log_e 8$.

$\therefore \frac{1}{2}\log_e(v^2 + 4) = s + \frac{1}{2}\log_e 8 \Rightarrow s = \frac{1}{2}\left(\log_e(v^2 + 4) - \log_e 8\right)$

$= \frac{1}{2}\log_e\left(\frac{v^2+4}{8}\right)$.

When $v = 6$, $s = \frac{1}{2}\log_e\left(\frac{36+4}{8}\right) = \frac{1}{2}\log_e 5 = \log_e\sqrt{5}$. (= 0.8047m)

$\frac{dv}{dt} = v^2+4 \Rightarrow \int\frac{dv}{v^2+4} = \int dt \Rightarrow \frac{1}{2}\tan^{-1}\frac{v}{2} = t + k$.

But $v = 2$ when $t = 0 \Rightarrow \frac{1}{2}\tan^{-1} 1 = 0 + k \Rightarrow k = \frac{\pi}{8}$.

$\therefore \frac{1}{2}\tan^{-1}\left(\frac{v}{2}\right) = t + \frac{\pi}{8} \Rightarrow t = \frac{1}{2}\tan^{-1}\left(\frac{v}{2}\right) - \frac{\pi}{8}$

When $v = 6$, $t = \frac{1}{2}\tan^{-1}\left(\frac{6}{2}\right) - \frac{\pi}{8} = \frac{1}{2}\tan^{-1}3 - \frac{\pi}{8}$.

$= \frac{1}{2}(1{\cdot}249) - \frac{1}{8}(3{\cdot}142)$

$= 0{\cdot}62245 - 0{\cdot}3925 = 0{\cdot}23$ seconds.

4. (a) $(1+x^3)\frac{dy}{dx} = x^2y \Rightarrow \int\frac{dy}{y} = \int\frac{x^2dx}{1+x^3} \Rightarrow \log_e y = \frac{1}{3}\log_e(1 + x^3) + c$.

But $y = 2$ when $x = 1 \Rightarrow \log_e 2 = \frac{1}{3}\log_e 2 + c \Rightarrow c = \frac{2}{3}\log_e 2$.

$\therefore \log_e y = \frac{1}{3}\log_e(1 + x^3) + \frac{2}{3}\log_e 2$

$\Rightarrow 3\log_e y = \log_e(1 + x^3) + 2\log_e 2$

$\Rightarrow \log_e y^3 = \log_e(1 + x^3) + \log_e 4 = \log_e(4 + 4x^3)$

$\Rightarrow y^3 = 4 + 4x^3 \Rightarrow y = \sqrt[3]{4 + 4x^3}$

(b) $\frac{d^2s}{dt^2} = -\left(\frac{ds}{dt}\right)^2 \Rightarrow \frac{dv}{dt} = -v^2$ where $v = \frac{ds}{dt}$.

$\therefore \int v^{-2}\,dv = \int -dt \Rightarrow -\frac{1}{v} = -t + c$

But $v = 1$ when $t = 0 \Rightarrow -1 = 0 + c \Rightarrow c = -1$

$\therefore -\frac{1}{v} = -t - 1 \Rightarrow \frac{1}{v} = t + 1 \Rightarrow \boxed{v = \frac{1}{t+1}}$

$v = \frac{ds}{dt} = \frac{1}{t+1} \Rightarrow \int ds = \int\frac{dt}{t+1} \Rightarrow s = \log_e(t + 1) + k$.

But $s = 0$ when $t = 0 \Rightarrow 0 = \log_e(1) + k \Rightarrow k = 0$

$\therefore s = \log_e(t + 1)$.

This is the same question. When $t = 1$, $s = \log_e(1 + 1) = \log_e 2$.

5. (a) $\frac{d^2y}{dx^2} + \frac{2}{y^3} = 0 \Rightarrow \frac{d^2y}{dx^2} = \frac{-2}{y^3}$. Let $v = \frac{dy}{dx} \Rightarrow v\frac{dv}{dy} = -\frac{2}{y^3} \Rightarrow$

$\int v\, dv = \int -2y^{-3} dy \Rightarrow \frac{1}{2}v^2 = \frac{1}{y^2} + c$.

But $v = \sqrt{2}$ when $y = 1 \Rightarrow 1 = 1 + c \Rightarrow c = 0 \therefore \frac{1}{2}v^2 = \frac{1}{y^2} \Rightarrow \boxed{v = \frac{\sqrt{2}}{y}}$

($v = -\frac{\sqrt{2}}{y}$ won't work). $v = \frac{dy}{dx} = \frac{\sqrt{2}}{y} \Rightarrow \int y\, dy = \int \sqrt{2}\, dx \Rightarrow \frac{1}{2}y^2 = \sqrt{2}x + k$.

But $x = \sqrt{2}$ when $y = 1 \Rightarrow \frac{1}{2} = 2 + k. \Rightarrow k = -\frac{3}{2}$.

$\therefore \frac{1}{2}y^2 = \sqrt{2}x - \frac{3}{2} \Rightarrow y^2 = 2\sqrt{2}x - 3 \Rightarrow y = \sqrt{2\sqrt{2}x - 3}$.

(b) $\frac{d^2s}{dt^2} = 5 - v^2$. But $\frac{d^2s}{dt^2} = \frac{dv}{dt} = \left(\frac{dv}{ds}\right)\left(\frac{ds}{dt}\right) = v\frac{dv}{ds}$,

$\therefore v\frac{dv}{ds} = 5 - v^2 \Rightarrow \int \frac{v\, dv}{5 - v^2} = \int ds \Rightarrow -\frac{1}{2}\log_e(5 - v^2) = s + c$.

But $v = 0$ when $s = 0$ (starts from rest) $\therefore -\frac{1}{2}\log_e 5 = 0 + c$

$\Rightarrow c = -\frac{1}{2}\log_e 5. \therefore -\frac{1}{2}\log_e(5 - v^2) = s - \frac{1}{2}\log_e 5 \Rightarrow$

$s = \frac{1}{2}\log_e\left(\frac{5}{5 - v^2}\right)$. When $s = 1.5$, $1.5 = \frac{1}{2}\log_e\left(\frac{5}{5 - v^2}\right)$

$\Rightarrow \log_e\left(\frac{5}{5 - v^2}\right) = 3 \Rightarrow \frac{5}{5 - v^2} = e^3 \Rightarrow \frac{5 - v^2}{5} = e^{-3} \Rightarrow v^2 = 5(1 - e^{-3})$

$\Rightarrow v = \sqrt{5(1 - e^{-3})} = \sqrt{4.7510} = 2.179$ m/s.

6. (a) $\frac{dy}{dx} + y\,\mathrm{Cos}^3 x = 0 \Rightarrow \frac{dy}{dx} = -y^2\,\mathrm{Cos}^3 x$

$\Rightarrow \int y^{-2}\, dy = -\int \mathrm{Cos}^3 x\, dx = -\int \mathrm{Cos}^2 x\,\mathrm{Cos}\, x\, dx$

(using the substitution $u = \mathrm{Sin}\, x$, this becomes $-\int(1 - u^2)du = -u + \frac{1}{3}u^3 = -\mathrm{Sin}\, x + \frac{1}{3}\mathrm{Sin}^3 x$) $\therefore -\frac{1}{y} = -\mathrm{Sin}\, x + \frac{1}{3}\mathrm{Sin}^3 x + c$.

But $y = 2$ when $x = \frac{\pi}{6} \Rightarrow -\frac{1}{2} = -\frac{1}{2} + \frac{1}{24} + c \Rightarrow c = -\frac{1}{24}$

$\therefore -\frac{1}{y} = -\mathrm{Sin}\, x + \frac{1}{3}\mathrm{Sin}^3 x - \frac{1}{24} \Rightarrow \frac{1}{y} = \frac{24\mathrm{Sin}x - 8\mathrm{Sin}^3 x + 1}{24}$

$\Rightarrow y = \frac{24}{24\mathrm{Sin}\, x - 8\mathrm{Sin}^3 x + 1}$.

(b) $\frac{d^2y}{dx^2} = K\frac{dy}{dx} \Rightarrow \frac{dv}{dx} = Kv$ where $v = \frac{dy}{dx}$.

$\therefore \int \frac{dv}{v} = \int K\, dx \Rightarrow \log_e v = Kx + A \Rightarrow \boxed{v = e^{Kx+A}}$

$v = \frac{dy}{dx} = e^{Kx+A} \Rightarrow \int dy = \int e^{Kx+A}\, dx \Rightarrow y = \frac{1}{K}e^{Kx+A} + B$.

(c) $\frac{d^2s}{dt^2} = \frac{1}{2}\left(\frac{ds}{dt}\right) \Rightarrow \frac{dv}{dt} = \frac{1}{2}v$, where $v = \frac{ds}{dt} \therefore \int \frac{dv}{v} = \int \frac{1}{2}\, dt$

$\Rightarrow \log_e v = \frac{1}{2}t + c$. But $v = 3$ when $t = 0 \Rightarrow \log_e 3 = c \therefore \log_e v = \frac{1}{2}t + \log_e 3$

$\Rightarrow \log_e \frac{v}{3} = \frac{1}{2}t \Rightarrow \frac{v}{3} = e^{\frac{1}{2}t} \Rightarrow \boxed{v = 3e^{\frac{1}{2}t}}$.

$\frac{ds}{dt} = 3e^{\frac{1}{2}t} \Rightarrow \int ds = \int_4^5 3e^{\frac{1}{2}t}\, dt$ (Since the 5th second is from $t = 4$ to $t = 5$)

$\Rightarrow s = 6e^{\frac{1}{2}t}\Big|_4^5 = 6e^{2\frac{1}{2}} - 6e^2 = 6(e^{2\frac{1}{2}} - e^2)$ metres.

7. (a) $\mathrm{Sin}\, x\frac{dy}{dx} = y\,\mathrm{Cos}\, x \Rightarrow \int \frac{dy}{y} = \int \frac{\mathrm{Cos}\, x\, dx}{\mathrm{Sin}\, x}$.

(Using $u = \sin x$ this becomes $\int \frac{du}{u} = \log_e u = \log_e \mathrm{Sin}\, x$). $\therefore \log_e y = \log_e \mathrm{Sin}\, x + c$

But $y = 2$ when $x = \frac{\pi}{6} \Rightarrow \log_e 2 = \log_e \frac{1}{2} + c \Rightarrow c = \log_e 2 - \log_e \frac{1}{2}$

$\Rightarrow c = \log_e 2 - \log_e \frac{1}{2} = \log_e\left(\frac{2}{\frac{1}{2}}\right) = \log_e 4 \therefore \log_e y = \log_e \mathrm{Sin}\, x + \log_e 4$

$\Rightarrow \log_e y = \log_e 4\,\mathrm{Sin}\, x \Rightarrow y = 4\,\mathrm{Sin}\, x$.

(b) $F = ma \Rightarrow 40 - 3\sqrt{x} = 8\frac{d^2x}{dt^2} \Rightarrow \frac{d^2x}{dt^2} = 5 - \frac{3}{8}x^{\frac{1}{2}} \Rightarrow v\frac{dv}{dx} = 5 - \frac{3}{8}x^{\frac{1}{2}}$

$\Rightarrow \int v\, dv = \int(5 - \frac{3}{8}x^{\frac{1}{2}})\, dx \Rightarrow \frac{1}{2}v^2 = 5x - \frac{1}{4}x^{3/2} + c$.

But $v = 0$ when $x = 0$ (starts from rest), therefore $c = 0$.

$\therefore \frac{1}{2}v^2 = 5x - \frac{1}{4}x^{3/2} \Rightarrow v^2 = 10x - \frac{1}{2}x^{3/2}$

When $x = 100$, $v^2 = 10(100) - \frac{1}{2}(100)^{3/2} = 1000 - 500 = 500 \Rightarrow v = \sqrt{500}$ m/s.

When $v = 0$, $10x - \frac{1}{2}x^{3/2} = 0 \Rightarrow x(10 - \frac{1}{2}x^{\frac{1}{2}}) = 0 \Rightarrow x = 0$ or $10 - \frac{1}{2}x^{\frac{1}{2}} = 0$

starting off. $10 - \frac{1}{2}\sqrt{x} = 0$

$\Rightarrow \sqrt{x} = 20$

$\Rightarrow x = 400$ m.

(I have verified with the Department of Education that this is the solution desired).

8. (a) $\frac{dv}{dt} = 9 - kv = \int \frac{dv}{g - kv} = \int dt \Rightarrow -\frac{1}{k}\log_e(g - kv) = t + c$.

$\Rightarrow \log_e(g - kv) = -k(t+c) \Rightarrow g - kv = e^{-k(t+c)} \Rightarrow v = \frac{1}{k}(g - e^{-k(t+c)})$

As $t \to \infty$, $e^{-k(t+c)} \to 0 \therefore v \to \frac{1}{k}(g) = \frac{g}{k}$.

(b) $\frac{d^2s}{dt^2} = -k(\frac{ds}{dt})^2 \Rightarrow \frac{dv}{dt} = -kv^2$.

When $t = 0$, $s = 0$, $v = 20$; when $s = 100$, $v = 10$.

$\therefore \int v^{-2}\, dv = \int -k\, dt \Rightarrow -\frac{1}{v} = -kt + c$. But $v = 20$ when $t = 0 \Rightarrow c = -\frac{1}{20}$

$\therefore -\frac{1}{v} = -kt - \frac{1}{20} \Rightarrow \frac{1}{v} = kt + \frac{1}{20} \Rightarrow \boxed{v = \frac{20}{20kt + 1}}$

$\frac{ds}{dt} = \frac{20}{20kt+1} \Rightarrow \int ds = \int \frac{20}{20kt+1}\, dt \Rightarrow s = 20\left(\frac{1}{20k}\log_e(20k + 1)\right) + A$,

where A is a constant.

$\Rightarrow s = \frac{1}{k}\log_e(20kt + 1) + A$

But $s = 0$ when $t = 0 \Rightarrow 0 = 0 + A \Rightarrow A = 0 \therefore s = \frac{1}{k}\log_e(20kt + 1)$

and $v = \frac{20}{20kt+1}$. When $s = 100$, $v = 10$.

This gives the simultaneous equations: $\frac{1}{k}\log_e(20kt + 1) = 100$ ...... (Equation 1)

when $\frac{20}{20kt+1} = 10$ ...... (Equation 2).

Equation 2 gives $(20kt + 1) = 2$ ... Equation 3. Now Equation 1 reads:

$\frac{1}{k}\log_e 2 = 100 \Rightarrow k = \frac{1}{100}\log_e 2$. Putting this into equation 3 gives:

$\frac{1}{5}(\log_e 2)\, t + 1 = 2 \Rightarrow t = \frac{5}{\log_e 2} = \frac{5}{0.6931} = 7.2$ seconds.

CHAPTER 12. SIMPLE HARMONIC MOTION.

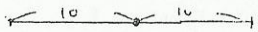
Exercise 12.A

1. (i) $F = k(l - l_o) = 9(3 - 2) = 9N.$

(ii) $F = k(l - l_o) = 9(5 - 2) = 27N.$

(iii) $F = 9(3\frac{1}{3} - 2) = 12N$

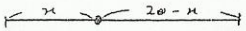
$24 = 9(l - 2) \Rightarrow l = 4\frac{2}{3}m.$

2. (i) 

$F_l = 2(10 - 1) = 18N.$

$F_r = 4(10 - 1) = 36N.$

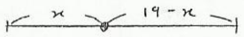
$\therefore F = F_r - F_l = 36 - 18 = 18N$

(ii) 

$F_l = 2(x - 1) = 2x - 2.$

$F_r = 4(20 - x - 1) = 76 - 4x.$

$F_l = F_r = 2x - 2 = 76 - 4x = x = 13m$ from left hand wall (LHW).

3. (i) 

$F_l = 5(x - 1) = 5x - 5.$

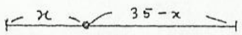
$F_r = 3(19 - x - 2) = 51 - 3x$

$F_l = F_r \Rightarrow 5x - 5 = 51 - 3x \Rightarrow x = 7m$ from LHW.

(ii) $F_l = 5x - 5.$

$F_r = 51 - 3x.$

$F_r - F_l = 16 \Rightarrow (51 - 3x) - (5x - 5) = 16 \Rightarrow x = 5m$ from LHW.

4. (i) 

$F_l = 7(x - 2) = 7x - 14.$

$F_r = 3(35 - x - 3) = 96 - 3x.$

$F_l = F_r \Rightarrow 7x - 14 = 96 - 3x \Rightarrow x = 11m$ from LHW.

(ii) $F_l = 7x - 14.$ $F_r = 96 - 3x.$

If the force is 40N to the right, then $F_r - F_l = 40$

$\Rightarrow (96 - 3x) - (7x - 14) = 40 \Rightarrow 110 - 10x = 40 \Rightarrow x = 7$

$\Rightarrow$ 7 metres from LHW.

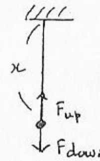
If the force is 40N to the left, then $F_l - F_r = 40 \Rightarrow (7x - 14) - (96 - 3x) = 40$

$\Rightarrow 10x - 110 = 40 \Rightarrow x = 15m$ from LHW.

5. $F = k(l - l_o) = 50(2-1) = 50$ N

This is the centripetal force and must equal $m\omega^2 r$.

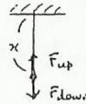
$\therefore m\omega^2 r = 50 \Rightarrow 1(\omega)^2 (2) = 50 \Rightarrow \omega = 5$ rads/sec.

6. 

$F_{up} = k(l - l_o) = 49(x - 1) = 49x - 49$ N.

$F_{down} = Mg = 10(9.8) = 98$ N.

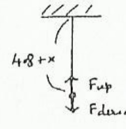
But $F_{up} = F_{down}$ (in equilibrium) $\therefore 49x - 49 = 98 \Rightarrow x = 3$ m.

7. 

$F_{up} = k(l - l_o) = 7(x - 2) = 7x - 14$ N,

$F_{down} = Mg = 2(9.8) = 19.6.$

$F_{up} = F_{down} \Rightarrow 7x - 14 = 19.6 \Rightarrow x = 4.8$ m.



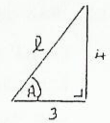
$F_{up} = k(l - l_o) = 7(4.8 + x - 2) = 19.6 + 7x.$

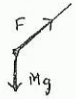
$F_{down} = Mg = 2(9.8) = 19.6$

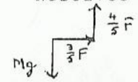
Nett force $= F_{up} - F_{down} = 19.6 + 7x - 19.6 = 7x$

8. (i) $l^2 = 4^2 + 3^2 \Rightarrow l = 5m$ $\therefore$ Extension $= 5 - 3 = 2m.$

(also $\cos A = \frac{3}{5}$, $\sin A = \frac{4}{5}$).



(ii) Forces 

Resolved 

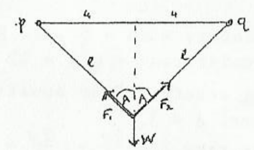
$F = k(l - l_o) = k(5 - 3) = 2k,$

1. $F_{up} = F_{down} \Rightarrow \frac{4}{5}F = Mg \Rightarrow \frac{4}{5}(2k) = Mg \Rightarrow k = \frac{5mg}{8}.$

2. Centripetal force $= m\omega^2 r \Rightarrow \frac{3}{5}F = m\omega^2 (3).$

But $F = 2k = s(\frac{5mg}{8}) = \frac{5mg}{4}$ $\therefore \frac{3}{5}(\frac{5mg}{4}) = m\omega^2 (3).$

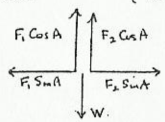
$\Rightarrow \omega^2 = \frac{g}{4} \Rightarrow \omega = \sqrt{\frac{g}{4}}$ rads/sec

9. 

$F_1 = k(l - l_o) = 10(l - 2) = (10l - 20)$ N

$F_2 = k(l - l_o) = 7(l - 1) = (7l - 7)$ N

Forces (Resolved)



9. contd ....

1. $F_{left} = F_{right} \Rightarrow F_1 \sin A = F_2 \sin A \Rightarrow F_1 = F_2 \Rightarrow 10\underline{1} - 20 = 7\underline{1} - 7$

$\Rightarrow \underline{1} = \frac{13}{3}$

Now $\sin A = \frac{opp}{hyp} = \frac{4}{13/3} = \frac{12}{13}$ $\therefore$ $\cos A = \frac{5}{13}$

2. $F_1 = 10\underline{1} - 20 = \frac{130}{3} - 20 = \frac{70}{3}$ N

$F_2 = F_1 = \frac{70}{3}$ N

$F_{up} = F_{down} \Rightarrow F_1 \cos A + F_2 \cos A = W \Rightarrow W = (\frac{70}{3})(\frac{5}{13}) + (\frac{70}{3})(\frac{5}{13}) = \frac{700}{39}$ N

EXERCISE 12.B
=============

1. (i) $\frac{2\pi}{7} = \frac{2\pi}{\omega} \Rightarrow \omega = 7$

Max. velocity $= \omega A = 7(5) = 35$ m/s

(ii) Max. acceleration $= \omega^2 A = (7)^2(5) = 245$ m/s^2

(iii) Total distance covered $= 4A = 20$m.

Time taken $= 2\pi/7$.

Average speed $= \frac{\text{Distance}}{\text{Time}} = \frac{20}{2\pi/7} = \frac{140}{2\pi} = \frac{70}{\pi}$ m/s.

2. $v^2 = \omega^2(A^2 - x^2)$

$x = \sqrt{7}$, $v = 9 \Rightarrow 81 = \omega^2(A^2 - 7)$ ......Equation 1

$x = 2$, $v = 6\sqrt{3} \Rightarrow 108 = \omega^2(A^2 - 4)$ .... Equation 2

Dividing equation 1 by 2 gives

$\frac{81}{108} = \frac{\omega^2(A^2 - 7)}{\omega^2(A^2 - 4)} \Rightarrow \frac{A^2 - 7}{A^2 - 4} = \frac{3}{4} \Rightarrow 3A^2 - 12 = 4A^2 - 28 \Rightarrow A = 4.$

Putting this result into equation 1 gives:

$81 = \omega^2(4^2 - 7) \Rightarrow \omega^2 = 9 \Rightarrow \omega = 3.$

Periodic time $T = \frac{2\pi}{\omega} = \frac{2\pi}{3}$ seconds

3. Max. velocity $= \omega A = 6$ .... Equation 1

Mac. acceleration $= \omega^2 A = 12$ .... Equation 2

Dividing equation 2 by equation 1 gives $\omega = 2$.

Therefore, $A = 3$.

Periodic time $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$ seconds.

To find a when $v = 2\sqrt{5}$:

Step 1 : Find x when $v = 2\sqrt{5}$:

$v^2 = \omega^2(A^2 - x^2) \Rightarrow 20 = 2^2(3^2 - x^2) \Rightarrow x = \pm 2.$

Step 2 : Find $\underline{a}$ when $x = \pm 2$.

$\underline{a} = -\omega^2 x = -(2)^2(\pm 2) = \pm 8$ m/s^2.

The magnitude of the acceleration is 8 m/s^2.

4. $x = 3 \sin 5t$. $\therefore \frac{dx}{dt} = 15 \cos 5t$. $\therefore \frac{d^2x}{dt^2} = -75 \sin 5t$

$= -25(3 \sin 5t) = -25x.$

Since the acceleration is proportional to x but in the opposite direction, it will perform SHM.

$x = 3 \sin 5t \Rightarrow 1.5 = 3 \sin 5t \Rightarrow \sin 5t = \frac{1}{2} \Rightarrow 5t = \frac{\pi}{6} \Rightarrow t = \frac{\pi}{30}$ seconds.

5. (i) $v^2 = \omega^2(A^2 - x^2)$

$v = 8$ when $x = 1 \Rightarrow 64 = \omega^2(A^2 - 1)$ .... Equation 1.

$v = 4$ when $x = 7 \Rightarrow 16 = \omega^2(A^2 - 49)$ .... Equation 2.

Dividing 1 by 2 gives $\frac{64}{16} = \frac{\omega^2(A^2 - 1)}{\omega^2(A^2 - 49)}$

$\Rightarrow \frac{A^2 - 1}{A^2 - 49} = \frac{4}{1} \Rightarrow 4A^2 - 196 = A^2 - 1 \Rightarrow A = \sqrt{65}$.

(ii) Putting this result into equation 1 gives:

$64 = \omega^2(65 - 1) \Rightarrow \omega = 1$. $\therefore$ $T = \frac{2\pi}{\omega} = \frac{2\pi}{1} = 2\pi$ seconds

(iii) When $x = 0$, $v^2 = \omega^2(A^2 - x^2) = 1(65 - 0) = 65$ $\therefore$ $v = \sqrt{65}$ cm/s.

6. (i) $x = 4 \cos 2t$. $\therefore \frac{dx}{dt} = -8 \sin 2t$. $\therefore \frac{d^2x}{dt^2} = -16 \cos 2t =$

$-4(4 \cos 2t) = -4x.$

Since the acceleration is proportional to the distance from p, but in the opposite direction, it will perform SHM ($A = 4$, $\omega = 2$)

(ii) Greatest distance $= A = 4$m

(iii) Its velocity is zero at the extreme point. Since the clock starts when the particle is at an extreme point, use $x = 4 \cos 2t$.

$x = 4 \cos 2t \Rightarrow 2.5 = 4 \cos 2t \Rightarrow \cos 2t = 0.625$

$\Rightarrow 2t = \cos^{-1}(0.625) \Rightarrow 2t = 0.8956$

$\Rightarrow t = 0.4478$ seconds.

7. $v^2 = \omega^2(A^2 - x^2)$. But $v = 24$ when $x = 5$. $\therefore$ $576 = \omega^2(A^2 - 25)$.... Equation 1

Also $\underline{a} = -\omega^2 x$. But $\underline{a} = -20$ when $x = 5$. ($\underline{a}$ and x are always of opposite sign)

$\therefore$ $-20 = -\omega^2(5) \Rightarrow \omega = 2$. Putting this into equation 1 gives:

$576 = 4(A^2 - 25) \Rightarrow A^2 - 25 = 144 \Rightarrow A = 13.$

Amplitude $= A = 13$. Periodic time $= \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$ seconds

In π seconds it performs 1 oscillation.

In 1 second it performs $\frac{1}{\pi}$ oscillations.

In 60 seconds it performs $\frac{60}{\pi} = 19.1$ oscillations.

Answer: 19 complete oscillations.

8. (i) $x = 13 \sin(\omega t + \epsilon)$. When $t = 0$, $x = 5 \Rightarrow 5 = 13 \sin \epsilon$

$\sin \epsilon = \frac{5}{13} = 0.3846 \Rightarrow \epsilon = \sin^{-1}(0.3846) = 0.3948.$

(ii) $v^2 = \omega^2(A^2 - x^2)$, $v = 24$ when $x = 5$. Also $A = 13$.

$\therefore$ $(24)^2 = \omega^2(13^2 - 5^2) \Rightarrow 576 = \omega^2(144) \Rightarrow \omega = 2.$

8. contd ...

(iii) $x = 0 \Rightarrow 13 \sin(\omega t + \epsilon) = 0 \Rightarrow \omega t + \epsilon = 0$ or π or 2π etc.

$\Rightarrow 2t + 0.3948 = 0$ or 3.1416 or 6.2832 etc.

The first time ($t > 0$) will be when $2t + 0.3948 = 3.1416 \Rightarrow 2t = 2.7468$

$\Rightarrow t = 1.3734$ seconds.

9. $v^2 = \omega^2(A^2 - x^2)$. $v = 3$ when $x = 1 \Rightarrow 9 = \omega^2(A^2 - 1)$ .. Equation 1

Also, $\underline{a} = \omega^2 x$. $\underline{a} = -3$ when $x = 1$. $-3 = -\omega^2(1) \Rightarrow \omega = \sqrt{3}$.

Putting this into equation 1 gives $9 = 3(A^2 - 1) \Rightarrow A^2 - 1 = 3 \Rightarrow A = 2$.

Maximum acceleration $= \omega^2 A = (3)(2) = 6$ m/s^2.

10. When $x = \sqrt{2}$, $a = -4\sqrt{2}$, $\therefore a = -\omega^2 x \Rightarrow -4\sqrt{2} = -\omega^2\sqrt{2} \Rightarrow \omega = 2$.

When $x = \sqrt{2}$, $v = 2$, and $\omega = 2$, $\therefore v^2 = \omega^2(A^2 - x^2) \Rightarrow 4 = 4(A^2 - 2) \Rightarrow$

$A = \sqrt{3}$.

Start the clock in the centre $\Rightarrow x = A \sin \omega t$ i.e. $x = \sqrt{3} \sin 2t$

To find t when $x = 1.5$: $1.5 = \sqrt{3}\sin 2t \Rightarrow 3 = 2\sqrt{3}\sin 2t$

$\Rightarrow \sin 2t = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\Rightarrow 2t = \frac{\pi}{3} \Rightarrow t = \frac{\pi}{6}$ seconds.

$F = Ma$. But $a = -\omega^2 x = -4(1.5) = -6$ and $m = 2$kg. $\therefore F = (2)(-6) = -12$ N

The force is of magnitude 12 N.

11. $x = a\cos(\omega t + \alpha) \Rightarrow \frac{dx}{dt} = -\omega a \sin(\omega t + \alpha) \Rightarrow \frac{d^2x}{dt^2} = -\omega^2 a\cos(\omega t + \alpha)$

$= -\omega^2 x$.

Since the acceleration is proportional to x but in the opposite direction, it will perform SHM.

$v = -2a$ when $x = \frac{3a}{5}$ and $A = a$. $v^2 = \omega^2(A^2 - x^2) \Rightarrow 4a^2 = \omega^2(a^2 - \frac{9a^2}{25})$

$\Rightarrow 4a^2 = \omega^2(\frac{16a^2}{25}) \Rightarrow \omega^2 = \frac{25}{4} \Rightarrow \omega = \frac{5}{2}$. Also $x = \frac{3a}{5}$ when $t = 0$.

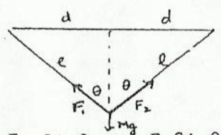
$\therefore x = a\cos(\omega t + \alpha) \Rightarrow \frac{3a}{5} = a\cos(\alpha) \Rightarrow \cos\alpha = \frac{3}{5} = 0.6 \Rightarrow \alpha = 0.9273$ radians.

Now, $a\cos(\omega t + \alpha) = 0 \Rightarrow \omega t + \alpha = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ or $\frac{5\pi}{2}$ etc.

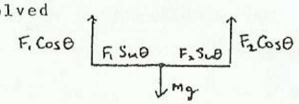
$\Rightarrow \frac{5}{2}t + 0.9273 = \frac{\pi}{2} = 1.571 \Rightarrow t = 0.2575$ seconds.

12.

Forces:



Resolved



$F_{left} = F_{right} \Rightarrow F_1 \sin\theta = F_2 \sin\theta \Rightarrow F_1 = F_2 \Rightarrow k_1(l - l_1) \Rightarrow k_2(l - l_2)$

$\Rightarrow k_1 l - k_1 l_1 = k_2 l - k_2 l_2 \Rightarrow l(k_1 - k_2) = k_1 l_1 - k_2 l_2 \Rightarrow l = \dfrac{k_1 l_1 - k_2 l_2}{k_1 - k_2}$

12. contd ..

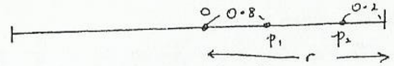
$\sin\theta = \frac{opp}{hyp} = \frac{d}{l} = \dfrac{d(k_1 - k_2)}{k_1 l_1 - k_2 l_2}$ q.e.d.

(b) Maximum acceleration must be not greater than g if the bodies are to stay on the platform.

$\Rightarrow \omega^2 A \leqslant 9.8 \Rightarrow \omega^2(0.2) \leqslant 9.8 \Rightarrow \omega \leqslant 7$. Taking ω at it's maximum value, 7. $T = \frac{2\pi}{\omega} = \frac{2\pi}{7} = 0.8977$ seconds

Number of oscillations per minute $= \frac{60}{0.8977} = 66$ complete oscillations

13.



As in question 11.

When $x = 0.8$, $v = 6$. When $x = r - 0.2$, $\underline{a} = -24$ (because $\underline{a}$ must be negative if x is positive)

$v^2 = \omega^2(A^2 - x^2)$. Put $x = 0.8$, $v = 6$, $A = r \Rightarrow 36 = \omega^2(r^2 - 0.64)$ .... Equation 1

$a = -\omega^2 x$. Put $x = r - 0.2$, $a = -24 \Rightarrow -24 = -\omega^2(r - 0.2)$

$\Rightarrow 24 = \omega^2(r - 0.2)$ ... Equation 2

Dividing equation 2 by equation 1 gives:

$\frac{24}{36} = \dfrac{\omega^2(r - 0.2)}{\omega^2(r^2 - 0.64)} \Rightarrow \dfrac{r - 0.2}{r^2 - 0.64} = \frac{2}{3}$

$\Rightarrow 2r^2 - 1.28 = 3r - 0.6 \Rightarrow 2r^2 - 3r - 0.68 = 0$,

$\Rightarrow 200r^2 - 300r - 68 = 0 \Rightarrow 50r^2 - 75r - 17 = 0 \Rightarrow (5r + 1)(10r - 17) = 0$

$\Rightarrow r = -0.2$ or $r = 1.7$. $r = -0.2$ has no meaning, so $r = 1.7$.

Putting this result into equation 2 gives: $24 = \omega^2(1.7 - 0.2) \Rightarrow 24 = \omega^2(1.5)$

$\Rightarrow \omega^2 = 16 \Rightarrow \omega = 4$.

Start clock at centre : $x = A\sin\omega t$ i.e. $x = 1.7\sin 4t$

At P_1, $x = 0.8 \Rightarrow 0.8 = 1.7\sin 4t \Rightarrow \sin 4t = \frac{8}{17} = 0.4706$

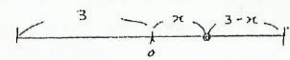
$\Rightarrow 4t = \sin^{-1}(0.4706) = 0.4900 \Rightarrow t = 0.1225$ seconds.

At P_2, $x = 1.5 \Rightarrow 1.5 = 1.7\sin 4t \Rightarrow \sin 4t = \frac{15}{17} = 0.8824$

$\Rightarrow 4t = \sin^{-1}(0.8824) = 1.0809 \Rightarrow t = 0.2702$ seconds.

Time to travel from P_1 to P_2 = 0.2702 − 0.1225 = 0.1477 seconds = 0.15 seconds

<u>EXERCISE 12.C</u>



1. $F_r = k(l - l_o) = 2(3 - x - 1) = 4 - 2x$.

$F_\ell = k(l - l_o) = 2(3 + x - 1) = 4 + 2x$.

$F = F_r - F_\ell = 4 - 2x - 4 - 2x = -4x$, $F = ma \Rightarrow -4x = 1(a) \Rightarrow a = -4x$

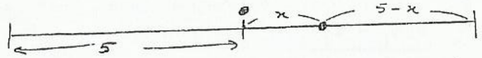
This is SHM with $\omega = 2$. Periodic time $= \frac{2\pi}{2} = \pi$ seconds

A = original distance from o = 1.

Midway between the walls $\Rightarrow x = 0 \Rightarrow v^2 = \omega^2(A^2 - x^2) \Rightarrow v^2 = 4(1 - 0) \Rightarrow$

$v = 2$ m/s.

2.



$F_r = k(l - l_o) = 9(5 - x - 1) = 36 - 9x$. $F_l = 9(5 + x - 1) = 36 + 9x$

$F_l = F_r - F_l = 36 - 9x - 36 - 9x = -18x$.

$F = ma \Rightarrow -18x = \frac{1}{2}a \Rightarrow a = -36x$.

It will perform SHM with $\omega = 6$. When it is released, $x = 2$. Therefore $A = 2$.

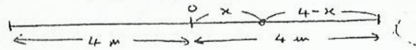
It starts from an extreme point. $\therefore x = a \cos \omega t$ i.e. $x = 2 \cos 6t$.

We want to find t when $x = 1$.

$\therefore 1 = 2 \cos 6t \Rightarrow \cos 6t = \frac{1}{2} \Rightarrow 6t = \frac{\pi}{3} \Rightarrow t = \frac{\pi}{18}$ seconds

$v^2 = \omega^2(A^2 - x^2) \Rightarrow v^2 = 6^2(2^2 - 1^2) = 108 \Rightarrow v = \sqrt{108} = 6\sqrt{3}$ m/s

3.



$F_r = 20(4 - x - 1) = 60 - 20x$, $F_l = 20(4 + x - 1) = 60 + 20x$.

$F = F_r - F_l = 60 - 20x - 60 - 20x = -40x$.

$F = ma \Rightarrow -40x = 5a \Rightarrow a = -8x$.

It will perform SHM with $\omega = \sqrt{8}$. When it is released, $x = 3$, therefore $A = 3$.

Maximum speed $= \omega A = \sqrt{8}\,(3) = 3\sqrt{8}$ m/s.

We want to find x when $v = \sqrt{8}$ m/s.

$v^2 = \omega^2(A^2 - x^2) \Rightarrow 8 = 8(9 - x^2) \Rightarrow 9 - x^2 = 1 \Rightarrow x = \sqrt{8}$ m.

It starts from an extreme point. $\therefore x = A\cos wt \Rightarrow x = 3 \cos \sqrt{8}t$

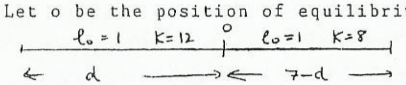
To find t when $x = \sqrt{8} \Rightarrow \sqrt{8} = 3 \cos \sqrt{8}t$

$\Rightarrow \cos \sqrt{8}t = \frac{\sqrt{8}}{3} = \frac{2.828}{3} = 0.9427$

$\Rightarrow \sqrt{8}t = \cos^{-1}(0.9427) = 0.34$.

$\Rightarrow t = 0.34/\sqrt{8} = 0.12$ seconds.

4. (i) Let o be the position of equilibrium



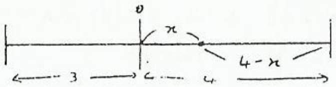
$F_r = k(l - l_o) = 8(7 - d - 1) = 48 - 8d$.

$F_l = k(l - l_o) = 12(d - 1) = 12d - 12$.

$F_r = F_l \Rightarrow 48 - 8d = 12d - 12 \Rightarrow d = 3$.

Answer: 3 metres from left hand wall.

(ii)



$F_r = k(l - l_o) = 8(4 - x - 1) = 24 - 8x$.

$F_l = k(l - l_o) = 12(3 + x - 1) = 24 + 12x$.

$F = F_r - F_l = 24 - 8x - 24 - 12x = -20x$.

$F = ma \Rightarrow -20x = 5a \Rightarrow a = -4x$. It will perform SHM with $\omega = 2$.

4. contd ...

(iii) When it is released its displacement, x, from o is $\frac{1}{2}$m.

Therefore $A = \frac{1}{2}$. Periodic time $= \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$ seconds

Maximum velocity $= \omega A = 2(\frac{1}{2}) = 1$ m/s.

(iv) Firstly, find x when $v = \frac{\sqrt{3}}{2}$:

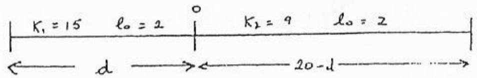
$v^2 = \omega^2(A^2 - x^2) \Rightarrow \frac{3}{4} = 4(\frac{1}{4} - x^2) \Rightarrow x = \frac{1}{4}$.

When $x = \frac{1}{4}$, $a = \omega^2 x = -(4)(\frac{1}{4}) = -1$ m/s^2.

The acceleration is of magnitude 1 m/s^2.

$F = ma \Rightarrow F = (5)(1) = 5$ N

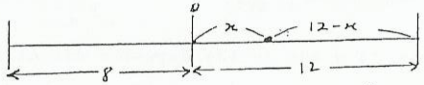
5.



$F_r = k(l - l_o) = 9(20 - d - 2) = 162 - 9d$.

$F_l = k(l - l_o) = 15(d - 2) = 15d - 30$.

$F_r = F_l \Rightarrow 162 - 9d = 15d - 30 \Rightarrow d = 8$.



$F_r = k(l - l_o) = 9(12 - x - 2) = 90 - 9x$.

$F_l = k(l - l_o) = 15(8 + x - 2) = 90 + 15x$.

$F = F_r - F_l = 90 - 9x - 90 - 15x = -24x$.

$F = ma \Rightarrow -24x = \frac{1}{6}a \Rightarrow a = -144x$.

It will perform SHM with $\omega = 12$

When it was released its displacement, x, from O was 1 m.
Therefore $A = 1$.

Maximum acceleration $= \omega^2 A = 144(1) = 144$ m/s^2.

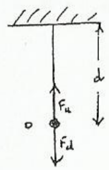
3/5 (Max acceleration) $= \frac{3}{5}(144) = \frac{432}{5}$ m/s^2

When $a = \frac{432}{5}$, what is x?. $a = -\omega^2 x \Rightarrow \frac{432}{5} = -144x \Rightarrow x = -\frac{3}{5}$

When $x = -\frac{3}{5}$, what is v? $v^2 = \omega^2(A^2 - x^2) \Rightarrow v^2 = 144(1 - \frac{9}{25}) \Rightarrow v^2 = 144\,(\frac{16}{25})$

$\Rightarrow v = 12(\frac{4}{5}) = 48/5 = 9.6$ m/s.

6.

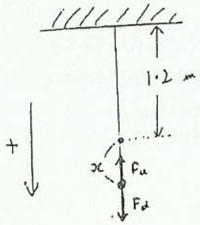


$F_d = Mg = 2(9.8) = 19.6$,

$F_u = k(l - l_o) = 98(d - 1)$.

$F_u = F_d \Rightarrow 98d - 98 = 19.6 \Rightarrow d = 1.2$m.

6. contd ....



$F_d = M_g = 2(9.8) = 19.6$.

$F_u = k(\frac{l}{} - \frac{l}{}o) = 98(1.2 + x - 1)$
$= 19.6 + 98x$

$F = F_d - F_u = 19.6 - 19.6 - 98x = -98x$

$F = ma \Rightarrow -98x = 2a \Rightarrow a = -49x$.

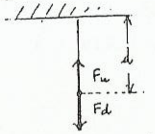
It will perform SHM with $\omega = 7$.

When it was released, it's displacement from o was 0.2m. Therefore, $A = 0.2$.

We start the clock at the centre. $\therefore$ $x = A \text{ Sin } wt \Rightarrow x = 0.2 \text{ Sin } 7t$

To find t when $x = \frac{\sqrt{2}}{10}$: $\frac{\sqrt{2}}{10} = 0.2 \text{ Sin } 7t \Rightarrow \text{Sin } 7t = \frac{1}{\sqrt{2}} \Rightarrow 7t = \frac{\pi}{4} \Rightarrow t = \frac{\pi}{28}$ seconds

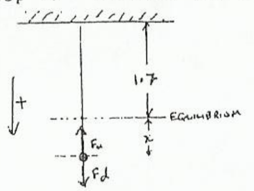
7. Step 1. To find position of equilibrium.



$F_d = Mg = \frac{1}{2}(9.8) = 4.9$. $\quad$ $Fu = k(\frac{l}{} - \frac{l}{}o) = 7(d - 1) = 7d - 7$.

$F_u = F_d \Rightarrow 7d - 7 = 4.9 \Rightarrow d = 1.7$

Step 2. Examine the forces when it is displaced a distance x from equilibrium.



$F_d = Mg = \frac{1}{2}(9.8) = 4.9$.

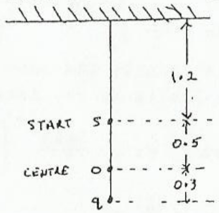
$F_u = k(1 - lo) = 7(1.7 + x - 1) = 4.9 + 7x$.

$F = F_d - F_u = 4.9 - 4.9 - 7x = -7x$.

$F = ma \Rightarrow -7x = \frac{1}{2}a \Rightarrow a = -14x$.

It will perform SHM with $\omega = \sqrt{14}$. It is released when it's displacement from equilibrium is 0.5m, therefore $A = 0.5$.

When it's distance below p is at a maximum it will be stationary. Therefore $v = 0$ m/s.



The journey to q from the starting position, s is divided into 2 parts: s to o and o to q.

s to o = $\frac{1}{4}$ of a full cycle

$\therefore$ Time $= \frac{1}{4}(\frac{2\pi}{\sqrt{14}}) = \frac{\pi}{2\sqrt{14}} = 0.42$ seconds.

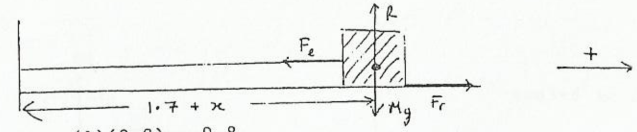
o to q: Start at centre $\therefore$ $x \Rightarrow A \text{ Sin } \omega t \Rightarrow x = 0.5 \text{ Sin } \sqrt{14}t$

$\Rightarrow 0.3 = 0.5 \text{ Sin } \sqrt{14}\, t \Rightarrow \text{Sin } \sqrt{14}t = 0.6 \Rightarrow \sqrt{14}t = \text{Sin}^{-1}(0.6) = 0.6435$

$\Rightarrow t = 0.6435/\sqrt{14} = 0.17$ seconds.

Total time = 0.42 + 0.17 = 0.59 seconds

8.



1. $R = Mg = (1)(9.8) = 9.8$.
2. $F_r = \mu R = \frac{1}{2}(9.8) = 4.9$.
3. $F_l = k(\frac{l}{} - \frac{l}{}o) = 7(1.7 + x - 1) = 4.9 + 7x$.

$F = F_r - F_l = 4.9 - 4.9 - 7x = -7x$

$F = Ma \Rightarrow -7x = 1a \Rightarrow a = -7x$

It will perform SHM with $w = \sqrt{7}$.

Periodic time $= \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{7}}$

The centre of oscillation, where $x = o$, is 1.7m from p. It was released where $x = 2$. Therefore $A = 2$.

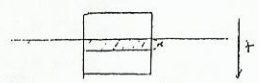
It starts from an extreme point $\therefore$ $x = A \text{ Cos } wt \Rightarrow x = 2 \text{ Cos } \sqrt{7}t$

To find t when $x = 0.3$ (i.e. 2 - 1.7)

$0.3 = 2 \text{ Cos } \sqrt{7}t \Rightarrow \text{Cos } \sqrt{7}t = 0.15$

$\Rightarrow \sqrt{7}t = \text{Cos}^{-1}(0.15) = 1.4202$

$\Rightarrow t = 1.4202/\sqrt{7} = 0.54$ seconds.



9. Mass $= V\rho = h^3(1000s) = 1000h^3 s$.

When it is depressed a distance x, the extra bouyancy B' is given by (note the downward is positive) B' = -weight of displaced liquid = $-V\rho g = (h^2x)(1000)g = -1000h^2 xg$

$F = ma \Rightarrow -1000h^2 xg = (1000h^3 s)a \Rightarrow a = \frac{-x}{hs}$ $\therefore$ It will perform SHM with

$\omega = \sqrt{\frac{g}{hs}}$. The periodic time $= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{hs}{g}}$

In this case $B' = -(h^2x)(1000k)g = 1000h'^2 xkg$.

$F = ma \Rightarrow -1000h^2 xkg = 1000h^3 sa \Rightarrow a = -\frac{gk}{hs}x$.

.. SHM with $\omega = \sqrt{\frac{gk}{hs}}$. Periodic time $= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{hs}{gk}}$.

10. As before, 0.6 of its height will be submerged i.e. (0.6)(80) = 48 cm.

Originally, its displacement from equilibrium is 2cm = 0.02m. Therefore $A = 0.02 = \frac{1}{50}$. Mass $= V\rho = [(0.8)(0.5)(0.2)]\,(600) = 48$ kg.

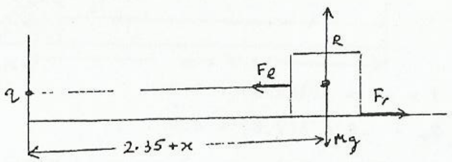
When it is displaced a distance x m below the water, the extra buoyancy B' is given by:

B' = weight of liquid displaced $= V\rho g = (0.5)(0.2)(x)(1000)(9.8) = -980x$

$F = ma \Rightarrow -980x = 48a \Rightarrow a = -\frac{980}{48}x = \frac{-245}{12}x$. This is SHM with $\omega = \sqrt{\frac{245}{12}}$.

Maximum acceleration $= \omega^2 A = \frac{245}{12} \times \frac{1}{50} = \frac{49}{120}$ m/s^2

11. First part: as before.



1. $R = Mg = 5(9.8) = 49$
2. $F_r = R = (1)(49) = 49$
3. $F_\ell = k(l - l_o) = 140(2.35 + x - 2) = 49 + 140x$

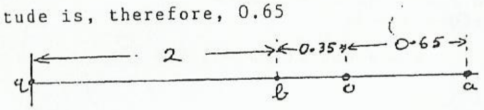
$F = F_r - F_\ell = 49 - 49 - 140x = -140x.$

$F = ma \Rightarrow -140x = 5a \Rightarrow a = -28x$.

This is SHM with $\omega = \sqrt{28} = 2\sqrt{7}$.

It starts when $|qp| = 3 \quad \therefore \quad 2.35 + x = 3 \Rightarrow x = 0.65$.

The ampitude is, therefore, 0.65



The journey from a to b can be divided into two parts: a to o and o to b.

a to o : $\frac{1}{4}$ of a full cycle $t = \frac{1}{4}\left(\frac{\pi}{\sqrt{7}}\right) = 0.2968$

o to b: $x = A\sin\omega t$ (starts at centre)

$0.35 = 0.65 \sin 2\sqrt{7}t$

$\Rightarrow \sin 2\sqrt{7}t = \frac{35}{65} = \frac{7}{13} = 0.5385.$

$\Rightarrow 2\sqrt{7}t = \sin^{-1}(0.5385) = 0.5687.$

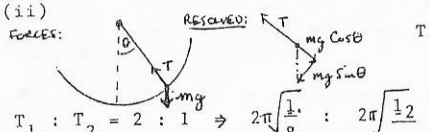
$\Rightarrow t = 0.1075$.

Total time = 0.2968 + 0.1075 = 0.4043 = 0.404 seconds

EXERCISE 12.D
==============

1. (i) $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{\frac{1}{2}}{9.8}} = 1.419$ s

(ii)



$T = mg\cos\theta = 2(19.6)(.99) = 19.404$ N

2. $T_1 : T_2 = 2 : 1 \Rightarrow 2\pi\sqrt{\frac{l_1}{g}} : 2\pi\sqrt{\frac{l_2}{g}} = 2 : 1 \Rightarrow \sqrt{l_1} : \sqrt{l_2} = 2 : 1$

$\Rightarrow l_1 : l_2 = 4 : 1$

3. $l_1 : l_2 = 4.9 \Rightarrow \sqrt{l_1} : \sqrt{l_2} = 2 : 3 \Rightarrow 2\pi\sqrt{\frac{l_1}{g}} : 2\pi\sqrt{\frac{l_1}{g}} = 2 : 3 \Rightarrow T_1 : T_2 = 2 : 3$

4. Let g' be the acceleration due to gravity at the satellite.

$g' < g$ (by Newtons Law) $\Rightarrow \frac{1}{g'} > \frac{1}{g} \Rightarrow 2\pi\sqrt{\frac{l}{g'}} > 2\pi\sqrt{\frac{l}{g}}$

$\Rightarrow$ it's period of oscilliation in longer than normal $\therefore$ it will go slow.

5. (i) Number of oscillations = $24 \times 60 \times 60 = 86400$.

(ii) The old periodic time = 1. $\therefore 2\pi\sqrt{\frac{l}{g}} = 1$

The new periodic time, $T = 2\pi\sqrt{\frac{l(1.02)}{g}} = \sqrt{1.02}\,(2\pi\sqrt{\frac{l}{g}}) = \sqrt{1.02}(1)$

= 1.01 seconds

It will now perform $\frac{86400}{1.01}$ oscillations in a day, i.e. 85545 oscillations in a day

It performs $(86400 - 85545) = 855$ fewer.

6. Let T = the original time = $\frac{60}{30}$ = 2 seconds

Let T' = the new time = $\frac{60}{31}$ seconds

$T' : T = \frac{60}{31} : 2 = 60 : 62 = 30 : 31 \quad \therefore \quad 2\pi\sqrt{\frac{l'}{g}} : 2\pi\sqrt{\frac{l}{g}} = 30 : 31$

$= \sqrt{l'} : \sqrt{l} = 30 : 31$

$\Rightarrow l' : l = 900 : 961 \Rightarrow \frac{l'}{l} = \frac{900}{961} \Rightarrow l' = \frac{900}{961} l = 0.93651$

$\therefore$ the % reduction = 100 - 93.65 = 6.35%

7. Let l', T' be the new length and periodic time respectively.

$l' : l = 2 : 1 \quad \therefore \quad T' : T = 2\pi\sqrt{\frac{l'}{g}} : 2\pi\sqrt{\frac{l}{g}} = \sqrt{2} : \sqrt{1} = 1.414 : 1$

$\therefore T' = 1.414\,T$

There has been an increase of 41% in its periodic time.

8. Let g' = acceleration due to gravity on the moon. l', T' will be the length and periodic time of the pendulum on the moon.

$T : T' = 2\pi\sqrt{\frac{l}{g}} : 2\pi\sqrt{\frac{l'}{g'}} = 2\pi\sqrt{\frac{8}{1}} : 2\pi\sqrt{\frac{3}{1/6}} = \sqrt{8} : \sqrt{18} = \sqrt{4} : \sqrt{9} = 2 : 3$

CHAPTER 13. RIGID BODY ROTATION.

Exercise 13. A

1. Let ρ = mass per unit length

$\therefore M = 2l\rho$

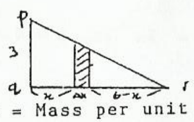
Also, $\Delta m = \rho(\Delta x) \quad \therefore \quad I = \sum(\rho\,\Delta x)x^2$

$\Rightarrow I = \int_0^{2l} \rho x^2\,dx = \frac{\rho x^3}{3}\Big|_0^{2l} = \frac{8\rho l^3}{3} = \frac{4ml^2}{3}$ (Since $M = 2l\rho$)

2. The solution is exactly the same as the proof of Theorem 13.3 except for the limits on the integration which, in this case are 0 and 2l.

$\therefore I = \int_0^{2l} 2\rho k x^2\,dx = \frac{2\rho k x^3}{3}\Big|_0^{2l} = \frac{16\rho k l^3}{3} = \frac{4(4\rho k l)l^2}{3} = \frac{4ml^2}{3}$, since $M = 4\rho kl$.

3.



Let ρ = Mass per unit area. $\therefore M = \frac{1}{2}(6)(3)\rho = 9\rho$

The length of the strip = $\frac{1}{2}(6-x) = 3 - \frac{1}{2}x$, by proportionality.

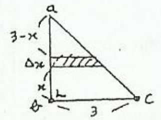
$\Delta M = \Delta x(3-\frac{1}{2}x)\rho \quad \therefore \quad I = \sum\Delta x(3-\frac{1}{2}x)\rho x^2 = \sum\Delta x(3x^2 - \frac{1}{2}x^3)\rho$

The values of x range from 0 to 6. $\therefore I = \int_0^6 (3x^2 - \frac{1}{2}x^3)\rho\, dx$

$= \rho(x^3 - \frac{x^4}{8})\Big|_0^6 = \rho(216 - 162) = 54\rho = 6(9\rho) = 6M$ since $M = 9\rho$.

The radius of gyration, k, is given by $k = \sqrt{\frac{I}{M}} = \sqrt{\frac{6M}{M}} = \sqrt{6}$.

4.



Let ρ = the mass per unit area $\therefore M = \frac{1}{2}(3)(3)\rho = \frac{9\rho}{2}$

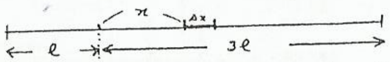
The length of the strip is $(3 - x)$, by proportionality $\therefore \Delta M = (3-x)(\Delta x)\rho$

$\therefore I = \sum(3-x)(\Delta x)\rho x^2 = \sum(3x^2 - x^3)\rho\,\Delta x$.

The values of x range from 0 to 3. $\therefore I = \int_0^3 (3x^2 - x^3)\,\rho\, dx =$

$\rho\,(x^3 - \frac{x^4}{4}) = \rho(27 - \frac{81}{4}) = \frac{27\rho}{4} = \frac{3M}{2}$, since $M = \frac{9\rho}{2}$

5.



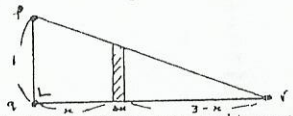
Let ρ = mass per unit length $\therefore M = 4l\rho$.

$\Delta M = (\Delta x)\rho \quad \therefore I = \sum(\Delta x)\rho x^2$ The values of x range from $-l$ to $+3l$

$\therefore I = \int_{-l}^{3l} \rho x^2\, dx = \frac{\rho x^3}{3}\Big|_{-l}^{3l} = (9\rho l^3) - (-\frac{\rho l^3}{3}) = \frac{28\rho l^3}{3} =$

$\frac{7(4l\rho)l^2}{3} = \frac{7ml^2}{3}$, since $M = 4l\rho$. $k = \sqrt{\frac{I}{M}} = \sqrt{\frac{7M\ell^2}{3M}} = \sqrt{\frac{7}{3}}\,\ell$.

6.



(i) Let ρ = mass per unit area. $\therefore M = \frac{1}{2}(3)(1)\rho = \frac{3}{2}\rho$.

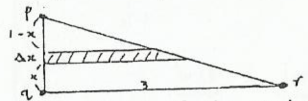
The length of the strip $= \frac{1}{3}(3-x)$, by proportionality $\therefore \Delta M = \frac{1}{3}(3-x)(\Delta x)\rho$

$I = \sum\frac{1}{3}(3-x)(\Delta x)\rho(x^2) = \sum\frac{1}{3}(3x^2 - x^3)\rho\,\Delta x$.

The values of x range from 0 to 3. $\therefore I = \int_0^3 \frac{1}{3}(3x^2 - x^3)\rho\, dx =$

$= \frac{1}{3}\rho\,(x^3 - \frac{x^4}{4})\Big|_0^3 = \frac{1}{3}\rho\,(27 - \frac{81}{4}) = \frac{9}{4}\rho = \frac{3}{2}(\frac{3}{2}\rho) = \frac{3}{2}M$, since $M = \frac{3}{2}\rho$.

(ii)



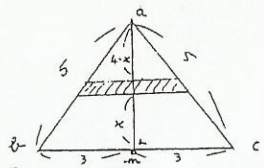
As before $M = \frac{3}{2}\rho$. The length of the strip $= 3(1-x) = 3 - 3x$.

$\therefore \Delta M = (3-3x)(\Delta x)\rho \quad \therefore I = \sum(3-3x)(\Delta x)\rho\, x^2 = \sum\rho\,(3x^2 - 3x^3)\Delta x$.

The values of x range from 0 to 1. $\therefore I = \int_0^1 \rho(3x^2 - 3x^3)dx =$

$\rho(x^3 - \frac{3x^4}{4})\Big|_0^1 = \rho(1 - \frac{3}{4}) = \frac{1}{4}\rho = \frac{1}{6}(\frac{3}{2}\rho) = \frac{1}{6}M$, since $M = \frac{3}{2}\rho$.

7.



Let m = mid-point of $[bc]$. $|am| = 4$, using Pythagoras' Theorem.

$M = \frac{1}{2}(6)(4)\rho = 12\rho$, where ρ = mass per unit area. Let l = the length of the strip.

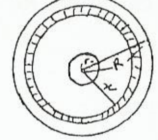
$\frac{l}{4-x} = \frac{6}{4}$, by proportionality $\therefore l = \frac{6}{4}(4-x) = 6 - \frac{3}{2}x$.

$\Delta M = (6 - \frac{3}{2}x)(\Delta x)\rho \quad \therefore I = \sum(6 - \frac{3}{2}x)(\Delta x)\rho x^2 = \sum(6x^2 - \frac{3}{2}x^3)\rho\,\Delta x$.

The values of x range from 0 to 4. $\therefore I = \int_0^4 (6x^2 - \frac{3}{2}x^3)\,\rho\, dx =$

$= \rho(2x^3 - \frac{3x^4}{8})\Big|_0^4 = \rho(128 - 96) = 32\rho = \frac{8}{3}(12\rho) = \frac{8}{3}M$, since $M = 12\rho$.

8.



Let ρ = mass per unit area. $\therefore M = (\pi R^2 - \pi r^2)\rho = \pi\rho(R^2 - r^2)$

The length of the strip $= 2\pi x \quad \therefore \Delta M = 2\pi x(\Delta x)\rho \quad \therefore I = \sum 2\pi x(\Delta x)\rho\, x^2$

The values of x range from r to R. $\therefore I = \int_r^R 2\pi\rho\, x^3\, dx = \frac{1}{2}\pi\rho x^4\Big|_r^R$

$= \frac{1}{2}\pi\rho(R^4 - r^4) = \frac{1}{2}\pi\rho(R^2 - r^2)(R^2 + r^2) = \frac{1}{2}M(R^2 + r^2)$ since

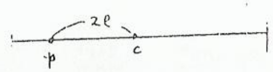
$M = \pi\rho(R^2 - r^2)$

EXERCISE 13.B

1. (i) $I = \frac{1}{3}(M)(3l)^2 = 3ml^2$.

   (ii) $I = \frac{4}{3}(M)(3l)^2 = 12ml^2$

   (iii)



$I_p = I_c + Md^2 = 3Ml + M(2l)^2 = 7Ml^2$

2. (a) (i) $I_A = \frac{1}{3}(M)l^2 = \frac{1}{3}Ml^2$

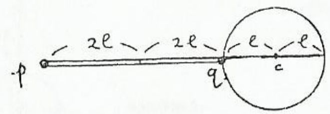
   (ii) $I_B = \frac{1}{3}(M)(2l)^2 = \frac{4}{3}Ml^2$

   (iii) $I_c = I_A + I_B$ (by perpendicular axis theorem) $= \frac{5}{3}Ml^2$

   (iv) $I_p = I_c + Md^2 = \frac{5}{3}Ml^2 + M(2l)^2 = \frac{17}{3}Ml^2$

   (b) Only (iv) and by $mr^2 = (3M)(2l)^2 = 12Ml^2$

3.



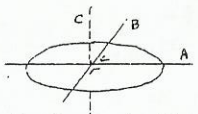
3. contd ..

ROD: $I_p = \frac{4}{3}(m)(2l)^2 = \frac{16}{3}ml^2 = 5\frac{1}{3}ml^2$

DISC: $I_p = I_c + md^2 = \frac{1}{2}(2m)(l)^2 + (2m)(5l)^2 = 51\,ml^2$

TOTAL $= 5\frac{1}{3}ml^2 + 51ml^2 = 56\frac{1}{3}ml^2$

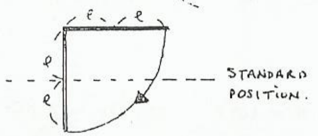
4.



By perpendicular axis theorem $I_A + I_B = I_C$

But since $I_A = I_B$ (by symmetry), $\therefore I_A = \frac{1}{2}I_C \quad \therefore I_A = \frac{1}{2}(\frac{1}{2}Mr^2) = \frac{1}{4}Mr^2$

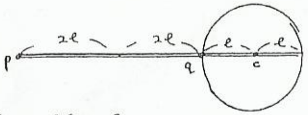
5. $I_p = \frac{4}{3}ml^2$



$mg\bar{h}_1 + \frac{1}{2}I\omega_1^2 = mg\bar{h}_2 + \frac{1}{2}I\omega_2^2 \Rightarrow mg(l) + \frac{1}{2}I(0) = mg(0) + \frac{1}{2}I\omega_2$.

$\Rightarrow I\omega_2^2 = 2mg \Rightarrow \frac{4}{3}ml^2\omega_2^2 = 2mg \Rightarrow \omega_2 = \sqrt{\frac{3g}{2l}}$

6. (a)



ROD: $I_p = \frac{4}{3}M(2l)^2 = \frac{16}{3}Ml^2$. DISC: $I_p = I_c + Md^2 = \frac{1}{2}Ml^2 + M(5l)^2 = \frac{51}{2}Ml^2$.

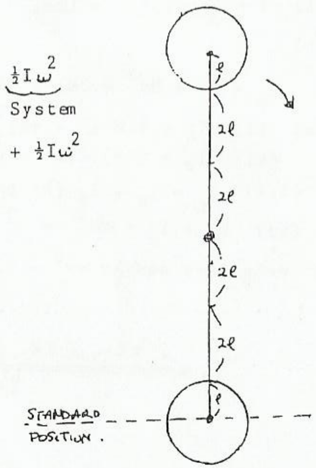
TOTAL $= \frac{16}{3}Ml^2 + \frac{51}{2}Ml^2 = \frac{185}{6}Ml^2$.

(b) $\underbrace{mg\bar{h}}_{\text{Rod}} + \underbrace{mg\bar{h}}_{\text{Disc}} + \underbrace{\frac{1}{2}I\omega^2}_{\text{System}} = \underbrace{mg\bar{h}}_{\text{Rod}} + \underbrace{mg\bar{h}}_{\text{Disc}} + \underbrace{\frac{1}{2}I\omega^2}_{\text{System}}$

$Mg(7l) + Mg(10l) + \frac{1}{2}I(0)^2 = Mg(3l) + Mg(0) + \frac{1}{2}I\omega^2$

$\Rightarrow I\omega^2 = 28Mgl \Rightarrow \frac{185}{6}Ml^2\omega^2 = 28\,Mgl \Rightarrow \omega = \sqrt{\frac{168g}{185l}}$

$\therefore v = \omega r = 5l\sqrt{\frac{168g}{185l}}$



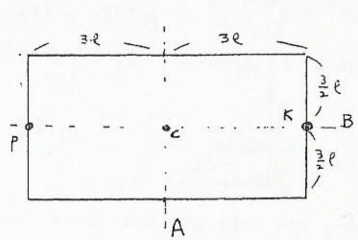
7.

The rectangular lamina: $I_A = \frac{1}{3}M(3l)^2 = 3Ml^2$

$I_B = \frac{1}{3}M(\frac{3}{2}l)^2 = \frac{3}{4}Ml^2$

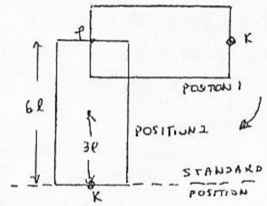
$I_c = I_A + I_B = \frac{15}{4}Ml^2$

$I_p = I_c + Md^2 = \frac{15}{4}Ml^2 + M(3l)^2 = \frac{51}{4}Ml^2$



The point mass : $I_p = mr^2 = M(6l)^2 = 36Ml^2$

The system $I_p = \frac{51}{4}Ml^2 + 36Ml^2 = \frac{195}{4}Ml^2$



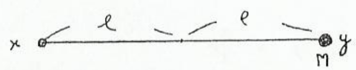
$\underbrace{Mg\bar{h}}_{\text{lamina}} + \underbrace{Mg\bar{h}}_{\text{point mass}} + \underbrace{\frac{1}{2}I\omega^2}_{\text{system}} = \underbrace{Mg\bar{h}}_{\text{lamina}} + \underbrace{Mg\bar{h}}_{\text{point mass}} + \underbrace{\frac{1}{2}I\omega^2}_{\text{system}}$

$Mg(6l) + Mg(6l) + \frac{1}{2}I(0)^2 = Mg(3l) + Mg(0) + \frac{1}{2}I\omega^2 \Rightarrow I\omega^2 = 18Mgl$

$\Rightarrow \frac{195}{4}Ml^2\omega^2 = 18\,Mgl \Rightarrow \omega = \sqrt{\frac{72g}{195l}} = \sqrt{\frac{24g}{65l}}$

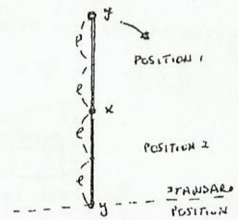
Speed of $k = \omega r = 6l\sqrt{\frac{24g}{65l}}$

8.



The rod: $I_x = \frac{4}{3}(3M)l^2 = 4Ml^2$. The point mass: $I_x = M(2l)^2 = 4Ml^2$

The system: $I_x = 8Ml^2$

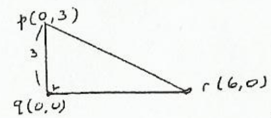


$\underbrace{Mg\bar{h}}_{\text{Rod}} + \underbrace{Mg\bar{h}}_{\text{Point Mass}} + \underbrace{\frac{1}{2}I\omega^2}_{\text{System}} = \underbrace{Mg\bar{h}}_{\text{Rod}} + \underbrace{Mg\bar{h}}_{\text{Point Mass}} + \underbrace{\frac{1}{2}I\omega^2}_{\text{System}}$

$(3M)g(3l) + Mg(4l) + \frac{1}{2}I(\frac{g}{l}) = (3M)gl + Mg(0) + \frac{1}{2}I\omega^2 \Rightarrow 9Mgl + 4Mgl + \frac{1}{2}(8Ml^2)(\frac{g}{l})$

$= 3Mgl + \frac{1}{2}(8Ml^2)\omega^2 \Rightarrow 4Ml^2\omega^2 = 14Mgl \Rightarrow \omega = \sqrt{7g/2l}$.

9. (a)

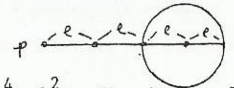


1. contd ......

The mass of the system is 2m.

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{16ml^2/3}{(2m)g\,(3l/2)}} = 2\pi\sqrt{\frac{16l}{9g}} = \frac{8}{3}\pi\sqrt{\frac{l}{g}}$$

2.

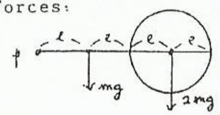


The rod: $I_p = \frac{4}{3}ml^2$. The disc: $I_p = I_c + md^2 = \frac{1}{2}(2m)l^2 + (2m)(3l)^2 = 19ml^2$

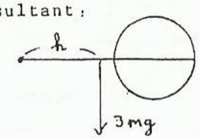
The system: $I_p = \frac{4}{3}ml^2 + 19ml^2 = \frac{61}{3}ml^2$

To find h:

Forces:



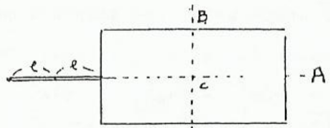
Resultant:



$mg(l) + 2mg(3l) = 3mg(h) \Rightarrow h = \frac{7}{3}l$

The mass of the system is 3m.

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{61ml^2/3}{(3m)g(7l/3)}} = 2\pi\sqrt{\frac{61l}{21g}}$$



3.

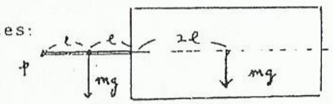
The rod: $I_p = \frac{4}{3}ml^2$. The lamina: $I_A = \frac{1}{3}ml^2$. $I_B = \frac{1}{3}m(2l)^2 = \frac{4}{3}ml^2$

$I_c = I_A + I_B = \frac{5}{3}ml^2$. $I_p = I_c + md^2 = \frac{5}{3}ml^2 + m(4l)^2 = \frac{53}{3}ml^2$

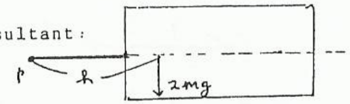
The system: $I_p = \frac{4}{3}ml^2 + \frac{53}{3}ml^2 = 19ml^2$

To find h:

Forces:



Resultant:



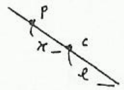
Taking moments about p: $mg(l) + mg(4l) = 2mgh \Rightarrow h = \frac{5}{2}l = 2\frac{1}{2}l$

The mass of the system is 2m.

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{19ml^2}{2mg(2\frac{1}{2}l)}} = 2\pi\sqrt{\frac{19l}{5g}}$$

If this is equal to $2\pi\sqrt{\frac{L}{g}}$, then $L = \frac{19}{5}l$

4.



$I_p = I_c + md^2 = \frac{1}{3}ml^2 + mx^2$.

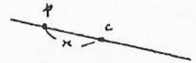
$h = x$. $m = m$.

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{\frac{1}{3}ml^2 + mx^2}{mgx}} = 2\pi\sqrt{\frac{7l}{6g}} \Rightarrow \frac{\frac{1}{3}l^2 + x^2}{x} = \frac{7l}{6}$$

$\Rightarrow 2l^2 + 6x^2 = 7lx \Rightarrow 6x^2 - 7lx + 2l^2 = 0 \Rightarrow (3x - 2l)(2x - l) = 0$

$\Rightarrow x = \frac{2}{3}l$ or $x = \frac{1}{2}l$.

5.



The rod: $I_p = I_c + md^2 = \frac{1}{3}ml^2 + mx^2$. The point mass: $I_p = (2m)x^2 = 2mx^2$

The system: $I_p = \frac{1}{3}ml^2 + 3mx^2$.

$h = x$.

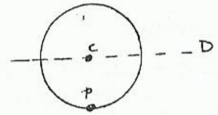
The total mass = 3m. $\therefore$ $T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{\frac{1}{3}ml^2 + 3mx^2}{(3m)gx}} = 2\sqrt{\frac{l^2 + 9x^2}{9gx}}$

$$\Rightarrow T^2 = 4\pi^2\left(\frac{l^2 + 9x^2}{9gx}\right) = \frac{4\pi^2}{g}\left(\frac{l^2 + 9x^2}{9x}\right)$$

$$\frac{d(T^2)}{dx} = \frac{4\pi^2}{g}\left(\frac{9x(18x) - (l^2 + 9x^2)(9)}{81x^2}\right) = 0 \Rightarrow 162x^2 - 9l^2 - 81x^2 = 0$$

$\Rightarrow 81x^2 = 9l^2 \Rightarrow x = \frac{1}{3}l$. To show that this is indeed a minimum show that $\frac{d^2(T^2)}{dx^2} > 0$ when $x = \frac{1}{3}l$.

6.

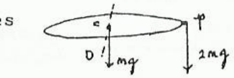


The disc: $I_D = \frac{1}{4}mr^2$. The point mass: $I_D = (2m)r^2 = 2mr^2$

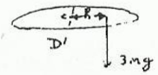
The system: $I_D = 2\frac{1}{4}mr^2$

To find h: examine the system tilted into a horizontal position

Forces



Resultant



Taking moments about D: $mg(0) + 2mg(r) = 3mgh \Rightarrow h = \frac{2}{3}r$.

The mass of the system is 3m.

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{2\frac{1}{4}mr^2}{(3m)g(2r/3)}} = 2\pi\sqrt{\frac{9r}{8g}}$$

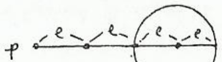
If this equals $2\pi\sqrt{\frac{l}{g}}$, then $l = 9r/8$.

1. contd ......

The mass of the system is 2m.

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{16ml^2/3}{(2m)g\,(3l/2)}} = 2\pi\sqrt{\frac{16l}{9g}} = \frac{8}{3}\pi\sqrt{\frac{l}{g}}$$

2.

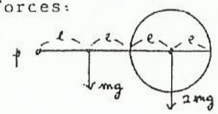


The rod: $I_p = \frac{4}{3}ml^2$. The disc: $I_p = I_c + md^2 = \frac{1}{2}(2m)l^2 + (2m)(3l)^2 = 19ml^2$

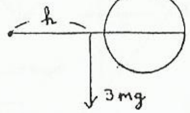
The system: $I_p = \frac{4}{3}ml^2 + 19ml^2 = \frac{61}{3}ml^2$

To find h:

Forces:



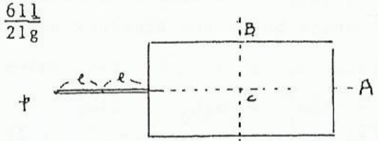
Resultant:



$$mg(l) + 2mg(3l) = 3mg(h) \Rightarrow h = \frac{7l}{3}$$

The mass of the system is 3m.

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{61ml^2/3}{(3m)g(7l/3)}} = 2\pi\sqrt{\frac{61l}{21g}}$$



3.

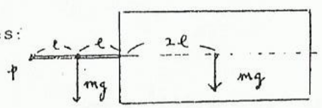
The rod: $I_p = \frac{4}{3}ml^2$. The lamina: $I_A = \frac{1}{3}ml^2$. $I_B = \frac{1}{3}m(2l)^2 = \frac{4}{3}ml^2$

$I_c = I_A + I_B = \frac{5}{3}ml^2$. $I_p = I_c + md^2 = \frac{5}{3}ml^2 + m(4l)^2 = \frac{53}{3}ml^2$

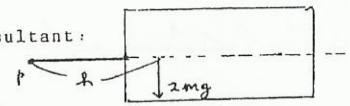
The system: $I_p = \frac{4}{3}ml^2 + \frac{53}{3}ml^2 = 19ml^2$

To find h:

Forces:



Resultant:



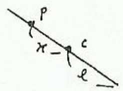
Taking moments about p: $mg(l) + mg(4l) = 2mgh \Rightarrow h = \frac{5}{2}l = 2\frac{1}{2}l$

The mass of the system is 2m.

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{19ml^2}{2mg(2\frac{1}{2}l)}} = 2\pi\sqrt{\frac{19l}{5g}}$$

If this is equal to $2\pi\sqrt{\frac{L}{g}}$, then $L = \frac{19}{5}l$

4.



$I_p = I_c + md^2 = \frac{1}{3}ml^2 + mx^2$.

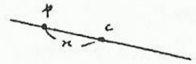
$h = x$. $m = m$.

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{\frac{1}{3}ml^2 + mx^2}{mgx}} = 2\pi\sqrt{\frac{7l}{6g}} \Rightarrow \frac{\frac{1}{3}l^2 + x^2}{x} = \frac{7l}{6}$$

$\Rightarrow 2l^2 + 6x^2 = 7lx \Rightarrow 6x^2 - 7lx + 2l^2 = 0 \Rightarrow (3x - 2l)(2x - l) = 0$

$\Rightarrow x = \frac{2}{3}l$ or $x = \frac{1}{2}l$.

5.



The rod: $I_p = I_c + md^2 = \frac{1}{3}ml^2 + mx^2$. The point mass: $I_p = (2m)x^2 = 2mx^2$

The system: $I_p = \frac{1}{3}ml^2 + 3mx^2$.

$h = x$.

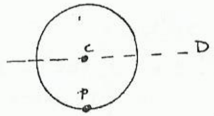
The total mass = 3m. $\therefore$ $T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{\frac{1}{3}ml^2 + 3mx^2}{(3m)gx}} = 2\sqrt{\frac{l^2 + 9x^2}{9gx}}$

$$\Rightarrow T^2 = 4\pi^2\left(\frac{l^2 + 9x^2}{9gx}\right) = \frac{4\pi^2}{g}\left(\frac{l^2 + 9x^2}{9x}\right)$$

$$\frac{d(T^2)}{dx} = \frac{4\pi^2}{g}\left(\frac{9x(18x) - (l^2 + 9x^2)(9)}{81x^2}\right) = 0 \Rightarrow 162x^2 - 9l^2 - 81x^2 = 0$$

$\Rightarrow 81x^2 = 9l^2 \Rightarrow x = \frac{1}{3}l$. To show that this is indeed a minimum show that $\frac{d^2(T^2)}{dx^2} > 0$ when $x = \frac{1}{3}l$.

6.

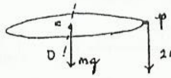


The disc: $I_D = \frac{1}{4}mr^2$. The point mass: $I_D = (2m)r^2 = 2mr^2$

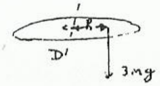
The system: $I_D = 2\frac{1}{4}mr^2$

To find h: examine the system tilted into a horizontal position

Forces



Resultant



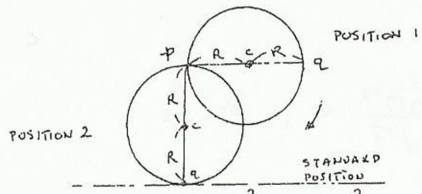
Taking moments about D: $mg(0) + 2mg(r) = 3mgh \Rightarrow h = \frac{2}{3}r$.

The mass of the system is 3m.

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{2\frac{1}{4}mr^2}{(3m)g(2r/3)}} = 2\pi\sqrt{\frac{9r}{8g}}$$

If this equals $2\pi\sqrt{\frac{l}{g}}$, then $l = 9r/8$.

7.



The disc: $I_p = I_c + Md^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$.

The point mass: $I_p = (2M)(2R)^2 = 8MR^2$.

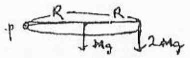
The system: $I_p = \frac{3}{2}MR^2 + 8MR^2 = \frac{19}{2}MR^2$.

$$\underbrace{mg\bar{h}}_{\text{Disc}} + \underbrace{mg\bar{h}}_{\text{Point Mass}} + \underbrace{\tfrac{1}{2}I\omega^2}_{\text{System}} = \underbrace{mg\bar{h}}_{\text{Disc}} + \underbrace{mg\bar{h}}_{\text{Point Mass}} + \underbrace{\tfrac{1}{2}I\omega^2}_{\text{System}}$$

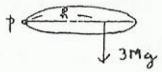
$$Mg(2R) + (2M)g(2R) + \tfrac{1}{2}I(0)^2 = Mg(R) + (2M)g(0) + \tfrac{1}{2}(\tfrac{19}{2}MR^2)\omega^2$$

$$5MgR = \frac{19}{4}MR^2\omega^2 \Rightarrow \omega = \sqrt{\frac{20g}{19R}}$$

To find h: FORCES:



RESULTANT:



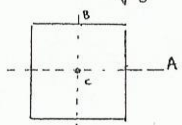
$$Mg(R) + 2Mg(2R) = 3Mg(h) \Rightarrow h = \tfrac{5}{3}R.$$

The mass of the system is 3M.

$$T = 2\pi\sqrt{\frac{I}{Mgh}} = 2\pi\sqrt{\frac{19MR^2/2}{(3M)g(5R/3)}} = 2\pi\sqrt{\frac{19R}{10g}}$$

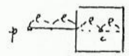
If this equals $2\pi\sqrt{\frac{l}{g}}$, then $l = \frac{19R}{10}$

8.



(i) $I_c = I_A + I_B = \frac{1}{3}Ml^2 + \frac{1}{3}Ml^2 = \frac{2}{3}Ml^2$

(ii)

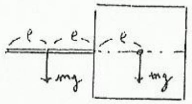


The rod: $I_p = \frac{4}{3}Ml^2$. The lamina: $I_p = I_c + Md^2 = \frac{2}{3}Ml^2 + M(3l)^2 = \frac{29}{3}Ml^2$.

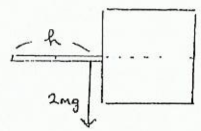
The system: $I_p = \frac{4}{3}Ml^2 + \frac{29}{3}Ml^2 = 11Ml^2$.

(iii)

Forces



Resultant



Taking moments about p: $mg(l) + mg(3l) = 2mgh \Rightarrow h = 2l$.

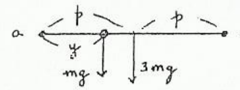
The mass of the system is 2m. $T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{11ml^2}{(2m)g(2l)}} = 2\pi\sqrt{\frac{11l}{4g}}$.

9. The rod: $I_a = \frac{4}{3}(3m)p^2 = 4mp^2$. The point mass: $I_a = my^2$

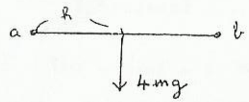
The system $= 4mp^2 + my^2$

To find h:

Forces



Resultant



Taking moments about $\underline{a}$:

$$mgy + 3mgp = 4mgh \Rightarrow h = \frac{y+3p}{4}$$

The mass of the system is 4m.

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{4mp^2 + my^2}{4mg\left(\frac{y+3p}{4}\right)}} = 2\pi\sqrt{\frac{4p^2 + y^2}{g(y+3p)}}$$

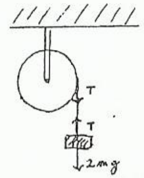
But this equals $2\pi\sqrt{\frac{40p}{33g}}$. $\therefore \frac{4p^2 + y^2}{y + 3p} = \frac{40p}{33}$

$$\Rightarrow 33y^2 - 40py + 12p^2 = 0 \Rightarrow (3y - 2p)(11y - 6p) = 0$$

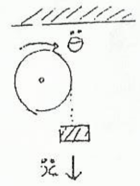
$$\Rightarrow y = \frac{2p}{3} \text{ or } \frac{6p}{11}.$$

10.

Forces:



Accelerations:



The 2m mass: $F = ma \Rightarrow 2mg - T = 2m\ddot{x}$ ..... equation 1

The disc: $L = I\ddot{\theta} \Rightarrow Tr = (\frac{1}{2}Mr^2)\ddot{\theta} \Rightarrow T = \frac{1}{2}mr\ddot{\theta} = \frac{1}{2}m\ddot{x}$, (Since $\ddot{x} = r\ddot{\theta}$).

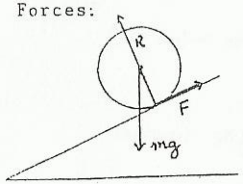
Putting this result into equation 1 gives:

$$2mg - \tfrac{1}{2}m\ddot{x} = 2m\ddot{x} \Rightarrow \ddot{x} = \tfrac{4}{5}g \quad \therefore \quad T = \tfrac{1}{2}m(\tfrac{4}{5}g) = \tfrac{2}{5}mg.$$

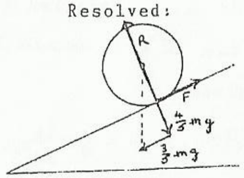
Answer: The tension is $\frac{2}{5}mg$ N; the acceleration $= \frac{4}{5}g$ m/s^2

11. $\tan A = \frac{3}{4} \Rightarrow \cos A = \frac{4}{5}$ and $\sin A = \frac{3}{5}$.

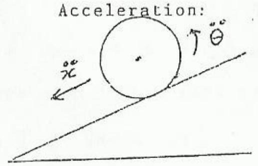
Forces:



Resolved:



Acceleration:



11. Contd ......

$R = \frac{4}{5}mg$ .... Equation 1

$\frac{3}{5}mg - F = m\ddot{x}$ ..... Equation 2

$Fr = (\frac{1}{2}mr^2)\ddot{\theta} \Rightarrow F = \frac{1}{2}mr\ddot{\theta} = \frac{1}{2}m\ddot{x}$ (since $\ddot{x} = r\ddot{\theta}$).

Putting this result into equation 2 gives:

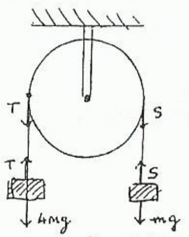
$\frac{3}{5}mg - \frac{1}{2}m\ddot{x} = m\ddot{x} \Rightarrow \ddot{x} = \frac{2}{5}g$ m/s^2

If it is on the point of slipping then $F = \mu R$.

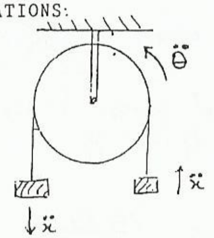
But $F = \frac{1}{2}m\ddot{x} = \frac{1}{2}m(\frac{2}{5}g) = \frac{1}{5}mg \therefore F = \mu R \Rightarrow \frac{1}{5}mg = \mu(\frac{4}{5}mg) \Rightarrow \mu = \frac{1}{4}$.

12.

FORCES :



ACCELERATIONS:



$4mg - T = 4m\ddot{x}$ ...... Equation 1

$S - mg = m\ddot{x}$ ...... Equation 2

$Tr - Sr = (\frac{1}{2}mr^2)\ddot{\theta} \Rightarrow T - S = \frac{1}{2}mr\ddot{\theta} = \frac{1}{2}m\ddot{x}$ (since $\ddot{x} = r\ddot{\theta}$)

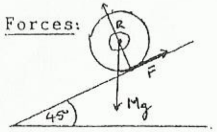
i.e. $T - S = \frac{1}{2}m\ddot{x}$ .... Equation 3

Adding all three equations gives: $3mg = 5\frac{1}{2}m\ddot{x} \Rightarrow \ddot{x} = 6g/11$

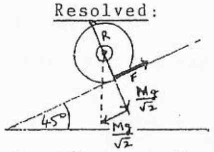
13. As exercise 13.A question 8, except with $R = 2$ and $r = 1$

$\therefore I = \frac{1}{2}m(4 + 1) = \frac{5}{2}m$.

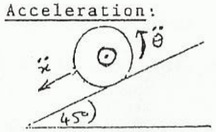
Forces:



Resolved:



Acceleration:



$R = \frac{Mg}{\sqrt{2}}$ ... Equation 1, $\frac{Mg}{\sqrt{2}} - F = M\ddot{x}$ ..... Equation 2

$F(2) = (\frac{5}{2}M)\ddot{\theta} \Rightarrow F = \frac{5}{4}M\ddot{\theta} = \frac{5}{8}M\ddot{x}$ (Since $\ddot{x} = 2\ddot{\theta}$, because the outer radius = 2)

Putting this result into equation 2 gives:

$\frac{Mg}{\sqrt{2}} - \frac{5}{8}M\ddot{x} = M\ddot{x} \Rightarrow \ddot{x} = \frac{8g}{13\sqrt{2}}$ m/s^2 $\therefore F = \frac{5}{8}M(\frac{8g}{13\sqrt{2}}) = \frac{5}{13\sqrt{2}}Mg$ Newtons.

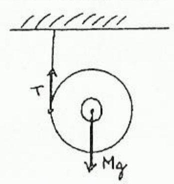
13. (contd)

If it is not slipping then $\mu \geq \frac{F}{R} \Rightarrow \mu \geq \frac{\frac{5}{13\sqrt{2}}Mg}{Mg/\sqrt{2}} \Rightarrow \mu \geq \frac{5}{13}$.

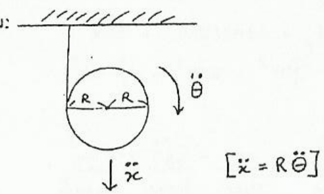
14.

As in Exercise 13.A question 8.

FORCES:



ACCELERATION:



$[\ddot{x} = R\ddot{\theta}]$

$F = ma \Rightarrow Mg - T = M\ddot{x}$ ...... Equation 1.

$L = I\ddot{\theta} \Rightarrow TR = \frac{1}{2}M(R^2 + r^2)\ddot{\theta} \Rightarrow TR^2 = \frac{1}{2}M(R^2 + r^2)(R\ddot{\theta})$

$\Rightarrow TR^2 = \frac{1}{2}M(R^2 + r^2)\ddot{x} \Rightarrow T = \frac{1}{2}M(\frac{R^2 + r^2}{R^2})\ddot{x}$

Putting this result into equation 1 gives:

$Mg - \frac{1}{2}M\left(\frac{R^2 + r^2}{R^2}\right)\ddot{x} = M\ddot{x} \Rightarrow Mg = M\ddot{x}\left(1 + \frac{R^2 + r^2}{2R^2}\right) \Rightarrow g = \ddot{x}\left(\frac{3R^2 + r^2}{2R^2}\right)$

$\Rightarrow \ddot{x} = \frac{2R^2g}{3R^2 + r^2}$ m/s^2 $\therefore T = \frac{1}{2}M\left(\frac{R^2 + r^2}{R^2}\right)\left(\frac{2R^2g}{3R^2 + r^2}\right)$

$= \frac{Mg(R^2 + r^2)}{3R^2 + r^2}$ N.

ADDENDA

Page 22: Exercise 3.A. Question 25, 26 and 27.

25. Since $\tan A = \frac{1}{2}$, $\cos A = \frac{2}{\sqrt{5}}$ and $\sin A = \frac{1}{\sqrt{5}}$.

Bullet: $u_x = 70\sqrt{5}\cos A = 140$ m/s. $u_y = 70\sqrt{5}\sin A = 70$ m/s.

$\therefore\ s_x = 140t \quad ; \quad s_y = 70t - 4.9t^2$.

Jet-fighter: $s_x = 140t \quad ; \quad s_y = 210$.

The bullet will strike the jet when $s_x = s_x$, and $s_y = s_y$.

$\Rightarrow\ 70t - 4.9t^2 = 210$

$\Rightarrow\ 4.9t^2 - 70t + 210 = 0$

$\Rightarrow\ 49t^2 - 700t + 2100 = 0$

$\Rightarrow\ 7t^2 - 100t + 300 = 0$

$\Rightarrow\ (7t - 30)(t - 10) = 0$

$\Rightarrow\ t = \frac{30}{7}$ or $t = 10$.

The bullet strikes first at $t = \frac{30}{7}$ seconds.

26. Bullet: $u_x = 35\cos A \quad ; \quad u_y = 35\sin A$

$\therefore\ s_x = 35\cos A\ t \quad ; \quad s_y = 35\sin A\ t - 4.9t^2$.

Bird: $s_x = 28t \quad ; \quad s_y = 5.6$.

(i) $s_x = s_x \Rightarrow 35\cos A\ t = 28t \Rightarrow \cos A = \frac{4}{5} \Rightarrow A = \tan^{-1}\frac{3}{4}$.

(Also $\sin A = \frac{3}{5}$)

(ii) $s_y = s_y$

$\Rightarrow\ 35\sin A\ t - 4.9t^2 = 5.6$

$\Rightarrow\ 21t - 4.9t^2 = 5.6$

$\Rightarrow\ 49t^2 - 210t + 56 = 0$

$\Rightarrow\ 7t^2 - 30t + 8 = 0$

$\Rightarrow\ (7t - 2)(t - 4) = 0$

$\Rightarrow\ t = 2/7$ or $t = 4$.

The bullet first strikes at $t = 2/7$ seconds.

27. $u_x = 35\cos A \quad ; \quad u_y = 35\sin A$.

$\therefore\ s_x = x = 35\cos A\ t$ ...equation 1.

$s_y = y = 35\sin A\ t - 4.9t^2$ ...equation2.

Equation 1 $\Rightarrow t = \frac{x}{35\cos A}$. Substituting this into equation 2 gives:

$$y = 35\sin A\left(\frac{x}{35\cos A}\right) - 4.9\left(\frac{x^2}{1225\cos^2 A}\right)$$

$$\Rightarrow\ y = x\tan A - \frac{x^2}{250\cos^2 A}$$

$$\Rightarrow\ 250y = 250x\tan A - \frac{x^2}{\cos^2 A}$$

But $\frac{1}{\cos^2 A} = \sec^2 A = 1 + \tan^2 A$.

$\therefore\ 250y = 250x\tan A - (1 + \tan^2 A)x^2$ ...qed.

If $x = 40$ when $y = 20$, then

$250(20) = 250(40)\tan A - (1 + \tan^2 A)(1600)$. Let $\tan A = T$, for short.

$\Rightarrow\ 5000 = 10000T - (1 + T^2)(1600)$

$\Rightarrow\ 50 = 100T - 16 - 16T^2$

$\Rightarrow\ 16T^2 - 100T + 66 = 0$

$\Rightarrow\ 8T^2 - 50T + 33 = 0$

$\Rightarrow\ (4T - 3)(2T - 11) = 0$

$\Rightarrow\ \tan A = \frac{3}{4}$ or $\tan A = \frac{2}{11}$.

If $\tan A = \frac{3}{4}$, then $\cos A = 4/5$ and so

$$t = \frac{x}{35\cos A} = \frac{40}{35(4/5)} = \frac{40}{28} = \frac{10}{7} \text{ seconds.}$$

If $\tan A = 11/2$, then $\cos A = \frac{2}{5\sqrt{5}}$ and so

$$t = \frac{x}{35\cos A} = \frac{40}{35(2/5\sqrt{5})} = \frac{20\sqrt{5}}{7} \text{ seconds.}$$

Page 70: Exercise 6.C, question 6.

6. $r_e : r_v = 150 \times 10^9 : 100 \times 10^9 = 3 : 2$.

$\therefore\ r_e^2 : r_v^2 = 9 : 4$.

Also, $m_e : m_v = 5 : 4$.

$$F_e : F_v = \frac{Gm_e m_s}{r_e^2} : \frac{Gm_v m_s}{r_v^2} = \frac{m_e}{r_e^2} : \frac{m_v}{r_v^2} = \frac{5}{9} : \frac{4}{4} = 5 : 9 .$$

Page 106: Exercise 9.B, question 4(b).

4.(b) Pressure under black oil = Pressure under oil

$\Rightarrow\ h(950)g = (0.08)(850)g$

$\Rightarrow\ h = 0.07158$ m $= 71.6$ cm.